MASTER'S THESIS

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Title: **Some mathematical programming-based models for a simplified evaluation of the capacity of railway networks**

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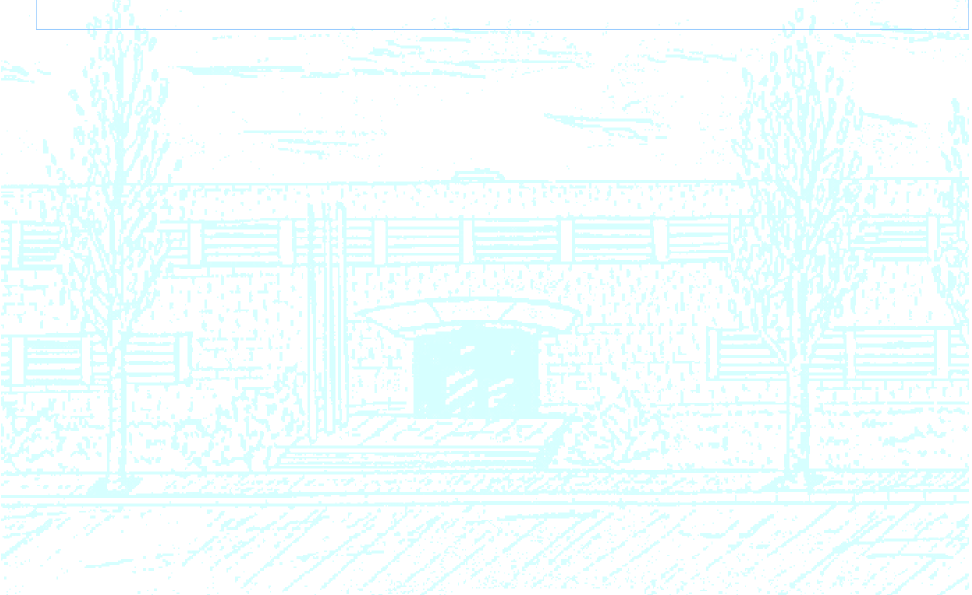

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Master Thesis

Some mathematical programming-based models for a simplified evaluation of the capacity of railway networks

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Doy las gracias a mi madre, padre y hermanos, por estar siempre apoyándome desde la distancia en cada uno de los proyectos en que me he embarcado. Agradecer también, a mis amigos, Grace y Alejandro por hacer del máster una gran experiencia. Al Dr. Esteve Codina por guiarme y apoyarme durante todo el desarrollo del trabajo. Finalmente, dedico un agradecimiento especial a mi novia Mar, que ha sido mi compañera ideal, en estos meses de trabajo.

Abstract

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The estimation of the available capacity in the rail networks is a truly relevant aspect that is part of decision processes both at an operational and strategic level of railway systems. The specific side conditions that are in force at any given moment can have a very strong impact in terms of the level of available capacity in a network. In a tactical decision-making environment, possibly many of these conditions can be well evaluated or known accurately. For example, a table of schedules for passenger services of different types can be known, which are the maintenance / inspection periods of the network and under these circumstances require an estimate of the maximum quantities of different types of circulations for freight trains. On the contrary, at different stages of the strategic process, such as the formation of new lines or the construction of new infrastructure, there may be a high level of uncertainty in relation to these additional factors, and even then an acceptable estimate of the capacity to accommodate services that the network can have, either for certain origin / destination relationships separately or working together in a certain scenario.

In this master's thesis, two similar models that have recently appeared in the literature aimed at establishing sets of constraints that incorporate basic variables of flow or number of circulations on elements of a railway network are analyzed and extended to the case of networks with a general configuration. The potential of these models consists of their ability to give estimates of maximum flows in the network taking into account only: a) their mutual interactions at specific points of the network, such as junctions or small stations, acting as the only "deterrent factors" that can be the agent that limits such flows, b) basic principles of blockade of railway sections for safety reasons. Also, these proposed models provide capacity estimates following different methodologies that allow finding either a rank of the maximum capacity or a pointwise estimate value; a remarkable characteristic is that the congestion level that may be reached in the network is included as part of the modelling process. Both approaches are not based on a scheduling methodology and it does not consider delays due to stochasticity or coming from queueing theory to compute the occupation percentage, but the time that a train could remain stopped at a node due to blocking.

The usefulness of the models that are the object of this master's thesis lies in their apparent simplicity and in their possibility of being included in more complex mathematical programming models oriented mainly to planning, without substantially increasing their complexity. The approaches are tested in two networks of different size to study the influence of the operational conditions, dwell times, headway time and infrastructure over a railway system.

Notation

$\mathbb{N}$	Node set
$\mathbb{C}$	Path set
ϕ	Set of origins and/or destinations
$\mathbb{R}$	Set of origin/destination pairs, where $\mathbb{R} = \{\ (p,q) : p,q \in \phi\ \}$
$\mathbb{A}$	Set of Arcs, where $\mathbb{A} = \{\ (i,j) : i,j \in \mathbb{N}\ \}$
$\mathbb{K}$	Set of train types
$\mathbb{G}$	Set of passing loops
$\mathbb{F}$	Set of passing loop arcs, $\mathbb{F} = \{\ (m,n) : m,n \in \mathbb{G}\ \}$
$\mathbb{D}$	Set of arcs within passing loop, $\mathbb{D} = \{\ (i,j) : i,j \in \mathbb{F}\ \}$
$\Omega_r[i,j]$	Set of path $r \in \mathbb{C}$ crossing arc $(i,j) \in \mathbb{A}$
$\xi_{p,q}[r]$	Path r that goes from the origin p to destination q
$\mathbb{S}_{i,j}[r]$	Set of arcs $(i,j) \in \mathbb{A}$ that use the path $r \in \mathbb{C}$
α_i^k	Time in the node i for the train type k
$\theta_{i,j}^k$	Time to cross the arc (i,j) with the train type k
$\rho_{p,q}^k$	Time to cross the path (p,q) with the train type k
$\vartheta_{i,j}^k$	Sum of dwell time at station i plus the time to cross the arc (i,j) for a train of type k, such that $(i,j) \in S_r$
$\gamma_{i,j,r}^k$	Enforced headway time for a train of type k, on arc (i,j) belonging to the path r
λ_r^k	Reduction factor in path r for train of type k
$\beta_{i,j}^k$	Headway time on arc (i,j) between traffic moving in opposite directions when there is no crossing loop at station j for train of type k.
$P_{i,j}^j$	Probability that the node $j \in \mathbb{N}$ is occupied
$\pi_{i,j}^j$	Occupancy probability for the remaining arcs (i',j) for a node $j \in \mathbb{N}$
$\mu_{i,j}^j$	Weighted probability that node $j \in \mathbb{N}$ is not occupied, on arch (i,j)
σ_r	Probability that the path r is not occupied
$\eta_{p,q}^k$	Total proportion of trains of type k on path (p,q) and (q,p)
$\nu_{p,q}^k$	Proportion of trains of type k on path (p,q)
T	Time period length
x_r^k	Number of trains of type k circulating on path r
$y_{i,j}^k$	Number of trains of type k circulating on arc (i,j)
$\widehat{x}_r^k$	Number of trains of type k circulating on path r not delayed
$\widehat{y}_{i,j}^k$	Number of trains of type k circulating on arc (i,j) delayed

Contents

Chapter 1
Introduction

1. Study context

Within the management of railway systems, one of the elements to take into account to provide an efficient train traffic, is the dimension of the network capacity. If this is a known factor, the amount of trains circulating can be handled and it is possible to know the available capacity, i.e. to know the numbers of additional trains that may be put into operative . At the same time, it allows the management operator to know if the line capacities are inefficient, because if there are more trains than the system allows, probably the network is under congestion or bottlenecks occur in some stations or node on the network, reducing the service level. On the other hand, having a correct estimation of the capacity, helps to know which are the volumes that can be moved along the network, and if at any time the demand is increased, know if it is necessary to expand the network to maintain the services in the estimated times. This last situation (expanding the network) is a really important issue to solve for the investors, because a good estimation of additional required capacity -Available capacity- could save a high investment. However, achieving a current approach of the capacity is a complex task for many elements that must be incorporated in the estimation model. Thus, as the railway knowing the capacity of a railway system produces a great impact on the service level, International Union of Railway (UIC from its French name, Union Internationale des Chemins de fer) has proposed a method (UIC Leaflet 405 OR, 1996) based on timetable to deal with this issue. Furthermore, other interested researchers have developed alternative methods that consider factors that are not incorporated in the methodology provided by UIC.

The influential parameters to estimate the railway capacity can be categorized depending on the literature. [7] indicates the balance between average speed, heterogeneity, stability and number of trains. On the other hand, [3] proposes to grouping the parameters under infrastructure, time and rolling stock. However, the real issue has been to find a common definition of what can be understood by the capacity of a railway network

- The capacity of a railway line is the ability to operate trains with an acceptable punctuality [8]

* The theoretical capacity is defined as being the maximal number of trains that can be operated on a railway link [9]
* Capacity is the measure of the ability to move a specific amount of traffic over a defined rail line with a given set of resources under a specific service plan [10]
* The only true measure of capacity, therefore, is the range of timetables that the network could support, tested against future demand scenarios and expected operational performance [11]
* The goal of capacity analysis is to determine the maximum number of trains that would be able to operate in a given railway infrastructure, during a specific time interval, given the operational conditions [3]

As can be seen in the aforementioned definitions, there are some differences between one definition or another. Nevertheless, in some cases the capacity will also be influenced by external factors such as demand requirement, weather conditions, or emergency situations. Hence, the methodology applied will depend on what has been adopted. Given this context, the present work will take up the definition given by [3] taking into account as much as possible the number of parameters that may impact in the capacity of a railway system.

Different types of capacity can be distinguished in a railway environment:

* *Theoretical capacity* which is associated with ideal conditions in the network use, namely, considering the time between each train and dwell times at minimum, neither is traffic variation considered and it assumes it being homogeneous. In other words, it is concerned with an upper bound that is almost impossible to achieve in normal conditions.
* *Practical capacity* is a more realistic measure of the capacity, being a percentage of between 60%-75% of the theoretical capacity according to [12], it includes an empirical mix of trains, timetables, traveling times and traffic variations.
* *Used capacity* is related with the current use of the network, and commonly, is lower than the *practical capacity.*
* *Available capacity* is used to indicate the amount of traffic that can be added to the railway system.

UIC 406 method leaflet published in 2004 [7] allows us to estimate the capacity consumption of a railway line in an efficient and fast way because its methodology is based on compressing the timetable as much as possible to take advantage of buffer times (i.e. the time when the line is not occupied). However, in order to apply the method and make it efficient, it is necessary to split the network into sections, because it is almost impossible to obtain useful results over a complete network without congestion problems due to the compression. Nevertheless, there is not a norm to choose the criteria to divide the network, as this will depend on the characteristics of the railway system. Furthermore, the methodology uses the timetable only as a base to estimate the capacity of the network and does not include other factors such as proportions of different types of trains that must be used. Thereby, to obtain the capacity through this method, it is necessary to have an efficient timetable, otherwise, the estimation of the capacity will be distorted.

An interesting approach is made by Olivera and E.,Smith in [5] using the job-shop scheduling problem on single track railway systems. That is achieved, considering the traffic of trains over a railway path as a *job*, that will be synchronized in time intervals depending on the capacity of the resource in a corridor or *line production* (in original job-shop scheduling problem). The stations with passing loops are the resources which allow the traffic in one direction or another, thus avoiding the use of the sections with two trains in opposite directions. The approach includes four constraints that allow us to adapt the original problem; the first one related with the minimum dwell time in the stations when two trains are waiting, which must be specified by the operator. Secondly a constraint associated with a priority factor, where it is possible to decide the next train dispatched. Then, the model adds a *blocking* that restricts the traffic in a section, at a time interval when a train is circulating, and finally, a headway time constraint. Nevertheless, in spite of being a good approach and obtaining useful results, the model can not be used in more complex railway network configuration with different railway paths.

The work presented in this master thesis exhibits two different approaches to estimate the capacity on a railway system of general configuration and not limited to a single section or corridor. The first model is an extension of the formulation done by [2], and the second model is based on work [1]. Both develop formulations as general as possible that can be applied in a large-scale situation. To achieve this, the concept of configuration of train flows, dwell times, signals, priorities, delays and double / single track is included at a section level. The models can be comparable, since some constraints share the same structure in both cases. However, the main difference lies in how the conflicts at nodes or stations are addressed. Moreover, it is necessary to stress that the timetable is not included explicitly in the constraints, but rather it is introduced under the concept of an efficient traffic through the rolling stock or the headway time. For example, the most efficient traffic (good scheduling) will allow the train flows first in one way, and then, when the last train, in that direction, has finished, start the second group of trains in the opposite direction. Thus, the headway time for each train would be the minimum possible.

Finally, it must be remarked that the models developed in this master thesis (and also the models in [1] and in [2]) do not take into account crucial factors that, in practice, pose further limitation to the network capacity such as:

* Existing operating lives satisfying passenger and/or freight demand.
* Fleet or rolling stock size available for carrying out the services corresponding to the available capacity to be estimated.
* Circulations corresponding to empty wagons (critical factor for freight flows) or for maintenance / inspection proposes.
* Limits imposed by technological factors such as deterioration of tracks
* Capacity of the depots in maximum number of units which may be allocated.
* Also, delays produced by shunting operations in train formation.

2. Report structure

This report comprises five chapters and three appendixes which are structured as follows:

Chapter 2: introduces, in a summarized way, the main parameters that have a big influence in the capacity of railway system and the simplifications that will be taken onto account to develop the models.

Chapter 3: presents the two models on which the formulation proposed by this work is based, explaining with detail the constraints of each one of them and commenting which aspect will be considered for further analysis.

Chapter 4: exhibits two new approaches to determine the railway capacity, and explain in detail the main differences between them and their constraints, and which will be the solving methodology in case that non-linearities arise. Furthermore, two additional extensions are presented, and the shortest path algorithm used is explained to generate the data sets automatically for the large-scale problems.

Chapter 5: shows the results obtained after applying both formulations proposed in chapter 4 in two different networks, considering double and single tracks for each formulation. In addition, the solutions of the respective extensions to strengthen the results are commented.

Chapter 6: comments and compares the obtained results and presents a discussion about which improvements can be added to the models in future studies.

Appendix A: because some constraints have similar structure, this appendix contains the formulation for each of the seven variations of the two models proposed (without the extensions) in the chapter 4 to avoid confusion and improve the understanding.

Appendix B: possesses all the tables which are discussed in the chapter 5 related with the case study applied for a medium network size.

Appendix C: contains the Rodalies network of Catalunya and the tables results associated with this railway system.

Chapter 2
Concepts of a railway network

In the following chapter, the components of a railway system will be presented, along with their definitions and the role that they play when the capacity of the network is estimated. Additionally, the simplifications considered in this master thesis, for the proposed models, will be presented at the end of the chapter.

1. Infrastructure

Railway network infrastructure is one of the most relevant factors that must be taken into account to design a model of railway capacity, because the model for a single track and that for a double one are not the same, and of course, because the models are not equal to one including both types of tracks. On the other hand, the mix of stations, nodes, junctions and passing loops, and their amount and allocation across the network will have a high impact on the network capacity

Corridors, belong to the main component in a railway network because some corridors can have hundreds of travel kilometers joining different cities or towns far away from downtown. In other words, all cities or points of a railway network are united through corridors, which can overlap. i.e. track where two or more corridors pass. However, to make the transport service possible, it is necessary to split the corridors into lines, stations, sections, and other components to regulate the train frequency so as to avoid accidents and make the service as fast as possible. Fig 1 shows an example of corridors in a railway network.

Lines, a corridor can be composed by several service lines whose objective is to allow the connection between different points of a network, based on demand flows.

Interlocking points, are network points that join two or more sections and depending on their size, they receive specific name. The *junctions* the smallest interlocking where the train can wait in front of, but not within the junction because it is only a switch area. *Stations* are small interlocking points as well, with the difference that the train can wait inside. Some stations have an additional track to allow the pass of another train. Additionally they are usually used to delimit a section and to load or unload passengers or freight. *Nodes* are those larger interlocking used usually to start or finish a service, change or repair of machinery. They are a

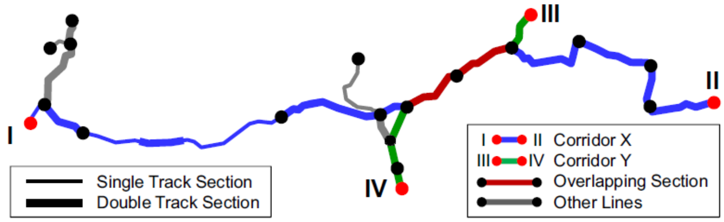

FIG. 1. Example of a railway infrastructure (Source: UIC 2004, ref [7])

complex network because they have several additional tracks due to in most cases, many sections converge there, and that forces the trains that are crossing the node to decrease their speed. *Crossing loop* or *Passing loop*, commonly used in single-tracks because they are network points with an additional track that allows the pass of another train in the opposite direction.

All Interlocking points described above, are conflict points in a railway network that reduce the capacity and they are factors to take into account to use in the best way to guarantee the smallest possible loss of capacity. In Fig. 2 some examples of interlocking are shown.

Signals are a mechanism of control that allows the flow of trains to circulate safely across the network since it indicates when the pass for a new section is enabled. Considering that in the cases when the device is in red, the train must reduce the speed, in theses situations, the capacity of the network is reduced as well.

Double and single track sections are relevant elements to determine the railway capacity. In the case when the railway has a section with double track, this will allow more flexibility to operate the train flows because it can move in each direction without the necessity of a crossing loop to wait for the pass of other trains in opposite direction. This factor is relevant because the capacity of a railway will be higher than in the case of single track. Particularly, it will be able to have a heterogeneous flow of trains and more or less timetable independence. On the other hand, a single track is a bit more complex since it allows the crossing of only one train at a time between two adjacent stations. Moreover, the running time between the stations must be at most half of the frequency, or in the case where they are not permitted to run as fast on the single-track section, the time that will be considered is the running time in crossing the section in both directions. This factor makes it a regular timetable and an homogeneous flow of trains necessary, and therefore, there will be less flexibility to move freight and passengers. Being more precise in terms of capacity, double track usually have around three to four times more capacity than single track configuration; however, four tracks rarely increase capacity by more than 50% over a double line [13].

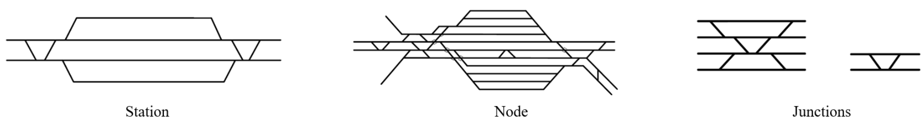

FIG. 2. Example of Interlocking (Source: UIC 2004, ref [7])

2. Time parameters

Time involved going through corridors or between two main stations is also an important factors that must be added into the model or should be indicated when simplified. Acceleration and deceleration periods are taken into account as lapses of time that in some cases are included in dwell time or in section running times. However, they will be important components in the case of studying big stations or complex nodes.

Timetable and priority, provide a strong mechanism for controlling the performance of the network, due to fixing an interval of service for each class of trains, Therefore, it is possible to reduce the bottleneck at interlockings, and in this way to get a better monitoring of the network. However, the priority factor plays an important role as traffic parameters, because if, the amount of prioritized trains is high, the capacity will be reduced on the network. Thus, an efficient timetable allows reducing the number of trains with priority and increases the service on the network.

Headway time, is defined as the long time between two consecutive trains. However, that definition will be used in the case where the train is on a double track, but in the cases of the single track, the headway time, for a train that is going to use the section bounded by crossing loops, will be the time it takes for another train, that is moving in opposite direction, to cross the section. In the Fig. 1 of Background formulation, It is possible to see that, the time that the train must wait to enter a single-track, (i.e., a section bounded by two crossing loops) will be the time it takes for the train that is traveling in the opposite direction to finish crossing the section.

Dwell time is the time that a train spends without moving. A usual example is when the train stops in a station to pick up or set down passengers and loading and unloading of freight. Therefore, as dwell time is longer, the standard capacity will be reduced considerably.

3. Rolling stock

This category is related with the train characteristics, either, its speed, length or transportation capacity. Thus, the composition of the types of trains (rolling stock), and how they are distributed in a railway system, has a high impact on the capacity on the network. Nevertheless, incorporation of this aspect on the model does not increase its complexity but, rather increases the amount of variables, and this can be an important issue in the solving time of the problem and could determine the solution method.

Heterogeneity in train types, usually different train types share all or part of a given railway network. For example, trains to travel short distances, stopping in each station and hence it will not need a high speed but probably will need a great capacity to transport all passengers. But if it is an express service, it will be a train operating at a higher speed. On the other hand, if the service is international, the trains must have a major capacity and in some case a high speed. In the case of freight trains there will also be a variety of train types depending on the service; rolling stock characteristics such as acceleration and deceleration are also important. Hence, the capacity of the network will depend on the proportion of each kind of train being used to support the corresponding demand.

4. Simplifications

The formulations presented in this master thesis include the elements which represent the main parameters that restrict the capacity in a railway system. However, some simplifications have been considered to carry out the models.

* The scheduling methodology has not been considered a base to develop the formulations.
* Double and single tracks have been taken into account separately, i.e. the network possesses only, double tracks or single tracks, but an explicit formulation with both cases at the same time has not been included .
* Headway time concept will depend on the infrastructure parameters. For instance, in single-track case, travel time between two crossing loops is considered, and for double track, the travel time between two node or stations independently if they possess a crossing loop.
* Mix of trains is considered as a fixed parameter given, and their difference lies in the speed of trains and not in the type of service. i.e., regional, international, urban or express service is not taken into account and neither is the concept of satisfying a minimum of passengers or freight demand included. Therefore, there is no difference between the type of service.
* The concept of priority is not included for the case when conflicts in some nodes appear.
* The data structure proposed allows the application in a design of large scale network, due to that it includes all the possible paths between origin / destination pairs.
* The dwell time is fixed and does not consider stochastic conflicts.
* Maneuver times have been reduced to the average travel speed on sections, and the deceleration and acceleration at the entrance or exit of a station has been considered in the dwell time.

Chapter 3
Background Formulations

In this chapter, the two models used as a basis for proposing the new approaches of the next chapter are explained. The first one will be "Techniques for absolute capacity determination in railways" by [1], and the second one, "An analytical approach to calculate the capacity of a railway system" by [2]. Moreover, for each one of these formulations, their components, simplifications and assumptions are highlighted and commented. However, it is necessary to mention that the nomenclature and notation of this reference articles have been modified in order to facilitate the comparison between them.

1. Techniques for absolute capacity determination in railways

This section presents a model which has as objective function, the maximization of the amount of trains on a railway system. To do this, the authors in [1] extend the bottleneck approach incorporating additional operational factors that the original (bottleneck approach) does not include such as the dwell times, the lengths of trains and stopping protocols. Also, the bottleneck analysis is usually used for a single train type, and as it was mentioned in the previous chapter, is a strong simplification that this approach considers in its formulation through the concept of *percentage train mix*, which it is computed from an actual or observed train mix.

Consequently, two distribution concepts are introduced:

* *Proportional distribution* ($\eta^k_{p,q}$) is the distribution of the types of trains, with regard to the total, through a corridor considering both direction.

* *Directional distribution*, ($\nu^k_{p,q}$) is the distribution of the types of trains, with regard to the total, through a corridor considering *only one direction.*

Thus, it is possible to incorporate the *composition of train types* in the model through the followings equations:

$$(1) \qquad x_r^k + x_{r'}^k = \eta_{p,q}^k \Bigg[\sum_{k' \in \mathbb{K}} (x_r^{k'} \ + \ x_{r'}^{k'}) \Bigg] \qquad \forall k \in \mathbb{K},\ \forall (p,q) \in \mathbb{R},\ \forall r \in \xi_{p,q}$$

$$(2) \qquad x_r^k = \nu_{p,q}^k [x_r^k \ + \ x_{r'}^k] \qquad \forall k \in \mathbb{K},\ \forall (p,q) \in \mathbb{R},\ \forall r \in \xi_{p,q}$$

Just to avoid confusion, it is explained that r' refers to the opposite path to r, i.e. if path $r \in \xi_{p,q}$, then r' is denoted as path $r' \in \xi_{q,p}$ such that it contains the same sections as r, but in reversed order.

Signals and reference locations are considered in the model to not overestimate the capacity of the network in the cases when bi-directional flows on a single track exist. To avoid this situation the *enforced headway* is used by the authors as proportional to the number of pairs of alternating trains. This will be dictated by a train schedule, and hence the absolute capacity will vary depending on the level of *fleeting*. Thus, an upper bound of the capacity will be the best sequence of trains, i.e. all trains in one direction, and the other fleet of trains in the opposite direction. With this schedule it is possible to decrease the enforced headway time on the system. On the other hand, when the traffic has the worst combinations, i.e. one train up, and the next down, and so on. The enforced headway time will be higher thus producing a lower bound capacity. Therefore, the equations to bound the problem in the corridors are:

$$(3) \qquad \sum_{k \in K} [\rho_{i,j}^k x_r^k + \rho_{j,i}^k x_{r'}^k] + \beta_{m,n}^k z \leq T \qquad \forall r \in \mathbb{C}$$

$$(4) \qquad \sum_{k \in K} [\rho_{i,j}^k x_r^k + \rho_{j,i}^k x_{r'}^k] + min(\beta_{m,n}^k, \beta_{n,m}^k) \leq T \qquad \forall r \in \mathbb{C}$$

It is necessary to highlight that both equations must be evaluated in the model in a separate way, i.e. solve the problem including first constraints (3) in order to obtain a lower bound, and then, separately solve the problem including only constraints (4) in order to obtain a upper bound. In this way, a range of values for the railway capacity will be obtained, and the actual level of capacity will then depend on the efficiency of the timetable that the operator will adopt in order to satisfy parameter demand

However, equations (3) and (4) are useful only when the problem is an evaluated problem at a corridor level. In the case when the situation requires a more exhaustive study of the situation in sections, the estimation of the headway time must be modified as the sum of two weighted average traveling times. In [1], the traveling times in particular were based upon the time it takes each train to reach the nearest crossing loop. Furthermore, the variable related with the number of trains traveling in the corridors, must be changed for a new variable associated with the number of trains traveling in the section. Thus, the new constraints to consider in the model are:

$$\sum_{k\in K}[\theta^k_{i,j}y^k_{i,j}+\theta^k_{j,i}y^k_{j,i}]+\beta^k_{i,j}(Y_{i,j},Y_{j,i})\leq T \qquad \forall r\in\mathbb{C},\ \forall(i,j)\in\mathbb{A} \tag{5}$$

$$\beta^k_{i,j}(Y_{i,j},Y_{j,i})=\gamma^k_{r,i,j}\left(\frac{x^k_r}{Y_{i,j}}\right)+\gamma^k_{r',j,i}\left(\frac{x^k_{r'}}{Y_{j,i}}\right) \qquad \forall(i,j)\in\mathbb{A},\forall k\in\mathbb{K} \tag{6}$$

$$y^k_{i,j}=\sum_{r\in\Omega_{i,j}}x^k_r \qquad \forall(i,j)\in\mathbb{A} \tag{7}$$

$$Y_{i,j}=\sum_{k\in\mathbb{K}}y^k_{i,j} \qquad \forall(i,j)\in\mathbb{A} \tag{8}$$

In equation (5) it is possible to see that the authors include $\beta^k_{i,j}$, the enforced headway time for a train of type k on corridor (p,q) on section (i,j). And also $x^k_r/Y_{i,j}$ which is the proportion of the current configuration of train types. Moreover, with equations (5) and (6) the problem becomes non-linear.

Dwell times are added as reduction factors over the total number of trains on the network. The article shows three different approaches to determine it, and additionally, propose two ways of how can be used in the resolution methods.

1.1. The first approach for the reduction factor. Is computed as the proportion of time that the train is delayed due to its dwell time in the station. The following set of equations allows us to see clearly the way in which the authors deal with this subject.

$$\rho^k_r=\sum_{(i,j)\in\mathbb{S}_r}\theta^k_{i,j} \qquad \forall r\in\mathbb{C},\ \forall k\in\mathbb{K} \tag{9}$$

$$\vartheta^k_r=\sum_{(i,j)\in\mathbb{S}_r}(\theta^k_{i,j}+\alpha^k_j) \qquad \forall r\in\mathbb{C},\ \forall k\in\mathbb{K} \tag{10}$$

$$\lambda^k_r=\frac{\rho^k_r}{\vartheta^k_r} \qquad \forall r\in\mathbb{C},\ \forall k\in\mathbb{K} \tag{11}$$

$$\lambda_r=\sum_{k\in\mathbb{K}}\eta^k_{p,q}[\mu^k_r\lambda^k_r+\mu^k_{r'}\lambda^k_{r'}] \qquad \forall(p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q},\ \forall r'\in\xi_{q,p} \tag{12}$$

In first instance, they compute the total travelling time on corridors, then the dwell on each station on the corridor is added to obtain the reduction factor for each train in a corridor, dividing the travelling time in the path (p,q) over the total time, including the dwell time. Once that is achieved, this factor must be adjusted in both directions of travel, through the directional distribution. And finally, we add up these factors for each type of train multiplied by its proportional distribution.

Once the reduction factor for each corridor has been obtained, this can be added directly from the objective function, after the initial problem (i.e. only considering the mixed train and signals constrains) has been resolved. Or as a second option, it can be resolved as one problem including the set of equations into the initial problem.

$$Absolute\ Capacity = \sum_{r\in\mathbb{C}} \lambda_r \sum_{k\in\mathbb{K}} x_r^k \tag{13}$$

1.2. The second approach for the reduction factor. Uses the same solving methodology, however, the set of equations is different. In this case, the sectional running time is multiplied by its respective direction proportion sum in both directions, to obtain the sectional running time on the corridor for each type of train. Likewise, the dwell times are obtained, then each factor is multiplied by its proportional distribution summing for all types of trains, getting the weighted average transit time and the weighted average total dwell time. And finally, the reduction factor will be the quotient between the last parameters obtained.

$$\alpha_r^k = \mu_{p,q} \sum_{(i,j)\in\mathbb{S}_r} \alpha_j^k + \mu_{q,p} \sum_{(j,i)\in\mathbb{S}_{r'}} \alpha_i^k \qquad \forall (p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q},\ \forall r'\in\xi_{q,p} \tag{14}$$

$$\rho_r^k = \mu_{p,q}\rho_r^k + \mu_{q,p}\rho_{r'}^k \qquad \forall (p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q},\ \forall r'\in\xi_{q,p} \tag{15}$$

$$\widehat{\rho}_r^k = \sum_{k\in\mathbb{K}} \eta_{p,q}^k \rho_r^k \qquad \forall r\in\xi_{p,q} \tag{16}$$

$$\widehat{\alpha}_r^k = \sum_{k\in\mathbb{K}} \eta_{p,q}^k \alpha_r^k \qquad \forall r\in\xi_{p,q} \tag{17}$$

$$\lambda_r = \frac{\widehat{\rho}_r^k}{\widehat{\rho}_r^k + \widehat{\alpha}_r^k} \qquad \forall r\in\xi_{p,q} \tag{18}$$

1.3. The third approach for the reduction factor. Is quite different because until now, prior estimation has been at the corridor level, however when it is at the section level, the estimation of the reduction factor must be modified, and it will have the same structure as (6).

$$\lambda_{i,j}^k = \lambda_{r,i,j}^k\left(\frac{x_r^k}{Y_{i,j}}\right) + \lambda_{r',j,i}^k\left(\frac{x_{r'}^k}{Y_{j,i}}\right) \qquad \forall (i,j)\in\mathbb{A}, \forall k\in\mathbb{K} \tag{19}$$

It is necessary to distinguish that this reduction factor is for a section for all types of trains, and the prior approach is the reduction factor for corridors. Hence, to introduce this factor $\lambda_{i,j}^k$ on the formulation, the proportional distribution and directional distribution must be adapted for a section level as follows:

$$\eta_{i,j}^k = \frac{y_{i,j}^k + y_{j,i}^k}{Y_{i,j} + Y_{j,i}} \qquad \forall (i,j)\in\mathbb{A}, \forall k\in\mathbb{K} \tag{20}$$

$$\mu_{i,j}^k = \frac{y_{i,j}^k}{Y_{i,j}} \qquad \forall (i,j)\in\mathbb{A}, \forall k\in\mathbb{K} \tag{21}$$

Thus, the new reduction factor is added multiplying this new equation and then incorporated in the general formulation.

The model presented by [1] is in general a good approach because it considers three relevant factors like composition of train flows, signals and dwell times. Nevertheless, some elements are simplified, for example, a *steady state* is assumed (i.e. the time it takes for trains to reach a specific position prior to the time period). Neither does it take into account the scheduling, but only introduces it implicitly through the equations (4) and (5), assuming a given configuration of trains. Delays caused by multiple trains interactions are also not handled.

2. An analytical approach to calculate the capacity of a railway system

This paper in [2] proposes new concepts and a different way to estimate the absolute capacity in a complex node. Here, the objective function presented seeks the maximization of the amount of trains on the network, using a relatively simple model that allows very short computing times and is based on the work done by [1] and [14] that is the paper previously presented. However, the main difference with the other approach is that, here the model is on the complex node and this forces a new way to tackle the problem. Thus, the authors split a complex node in its three main infrastructure elements; simple nodes, lines and station, where usually in a real case a big station is a complex node. Hence, in these particular cases, this kind of approach is very useful, although, some different types of times must be incorporated to achieve an actual approach. In this way, the time as accelerated and decelerated is considered. Thereby, and using the constraints of time intervals present in [2], it is that the three equations associated for each element is introduced into the model.

$$\sum_{r\in\Omega_{i,j}}\sum_{k\in\mathbb{K}}(x_r^k\alpha_{j,k}^1+\widehat{x}_r^k\widehat{\alpha}_{j,k}^1)\leq T \qquad \forall j\in\mathbb{G} \tag{22}$$

$$\sum_{r\in\Omega_{i,l}}\sum_{k\in\mathbb{K}}(x_r^k\alpha_{l,k}^2+\widehat{x}_r^k\widehat{\alpha}_{l,k}^2)\leq T \qquad \forall l\in\mathbb{G} \tag{23}$$

$$\sum_{r\in\mathbb{C}}\sum_{k\in\mathbb{K}}(x_r^k+\widehat{x}_r^k)\leq C^l \qquad \forall(p,q)\in\mathbb{R} \tag{24}$$

Eq. (22) is associated to the station. Eq. (23), to the time involved in a simple node, and Eq. (24) with a little different structure, related with the line capacity. As it is possible to observe, a new variable is introduced in theses equations, to show the amount of trains delayed. Thus, this model considers two types of variables; one type associated to the number of trains which circulate on the network without interruptions, called *regular trains*, and a second type variables that represent the amount of trains that have been delayed due to conflicts on a simple node, named as *irregular trains*. However, to make the difference between these two types of trains, the fraction of time P_r^j that path $r\in\Omega_{i,j}$ is occupied, or equivalently, the probability that a train is in a conflict with flows in path r, can be expressed by the following equation:

$$P_r^j = \frac{1}{T}\sum_{k\in\mathbb{K}}(x_r^k\alpha_{j,k}^2 + \widehat{x}_r^k\widehat{\alpha}_{j,k}^2) \qquad \forall j\in\mathbb{G}, \forall r\in\Omega_{i,j} \tag{25}$$

Here is represented the probability of conflict in a specific node j, where both variables are considered with their corresponding dwell times on the node. In fact, for the regular trains, the dwell time, will be defined by the schedule. However, the time for the irregular train includes the maneuver time, accelerated, decelerated time, and in some cases, as with a single track, the time waiting for enabled track, which are estimated separately. To introduce the constraint associated to the irregular trains, the authors consider the following approach.

$$x_r^j = (x_r^k + \widehat{x}_r^j)\frac{\left[P_r^j \sum_{m\in\Omega_{i,j}} P_m^j\right]}{\left[1 - \sum_{m\in\Omega_{i,j}} P_m^j\right]} \qquad \forall j\in\mathbb{G}, \forall r\in\Omega_{i,j}, m\neq r \tag{26}$$

As per a type of train and for each path belonging to specific simple node the Irregular trains will be a percentage of the total amount of them. This percentage is the result of the product between the probability obtained in the Eq. (26) for a node, and the sum of probability to the rest of path minus the given node, and moreover dividing by how much of the available fraction of time that all trains belonging to the path under consideration use. Let us note that when the node is extremely congested (i.e. the sum for each path that converges in the node is equal to one) the probability of interference for a given path is equal to the sum of percentage of the period T used by all other paths. However, the model will likely produce null values using the previous constraints, and to avoid this result, a maximum and minimum number of trains must be imposed for each type of them.

$$\sum_{k\in\mathbb{K}}(x_r^k + \widehat{x}_r^k) \geq LB_r \qquad \forall r\in\mathbb{C} \tag{27}$$

$$\sum_{k\in\mathbb{K}}(x_r^k + \widehat{x}_r^k) \leq UB_r \qquad \forall r\in\mathbb{C} \tag{28}$$

Furthermore, Eq. (26) makes the problem nonlinear, and the method that the authors in [2] propose to solve the model as a linear problem is the following:

step 1 fix a value for the matrix P
step 2 relax constraints (25)
step 3 solve the linear programming problem
step 4 insert the values x and $\widehat{x}$ in Eq. (25)
step 5 if $max_{r,k}\varepsilon_{r,k} \leq \varrho$ **stop**
step 6 else calculate the new values of P and go to step 2

Models proposed by [2] may have problems when applied on a large network with many stations, nodes, signals and single or double tracks. In other words, and based on the article presented above, the variables considered are only at corridor level and do not include the situation of what happens in the sections with the signals. Moreover, the objective of the model is not replacing the simulation or the scheduling approach which instead can take into account time evolution and crossed statistical dependence of the arrival characteristic of each type of train, neither does it include the proportional distribution of trains on the network. While it is true that, the diversity of train types (i.e. different speed and length) is added in the approach, the proportion of how this diversity is distributed on the network is not represented in any equation.

Chapter 4
Two new approaches

This chapter presents two new extended approaches of the models presented previously, where each one of them includes, in its formulation, the parameters related with configuration of train flows, signals, dwell time, headway time, traffic at section level and the corresponding variations for networks with double and single track. Both models will be explained separately with their respective linear and non-linear problem. The original formulation includes non-linear constraints, and to solve an alternative way through a heuristic method (fixed-point method), the linearization of the problem will be developed. Therefore, each approach contains: a non-linear problem with a set of general constraints and a variation for double and single track, together with a linear problem that includes the same contents. In Appendix A, the models proposed are stated for each one of the seven cases set out in this chapter. In addition, at the end of the chapter, two different extensions are presented that can be applied to the models proposed. Moreover, a methodology to automatically generate the sets of data through the shortest path algorithm is presented in the final subsection.

1. A fixed-point heuristic method for solving non-linear approach 1

Before explaining the model based on [2], it is necessary to describe some sets and subsets that will be used in almost every constraint, and whose understanding is important to observe the difference between a single and double track network system. For this purpose, let us call the set $\mathbb{G}$ a subset of $\mathbb{N}$, where $\mathbb{G}$ corresponds to the nodes with crossing loop. Let $\mathbb{F}$ be all sub-paths bounded by two nodes $(m, n) : m, n \in \mathbb{G}$. Let $\mathbb{D}$ set be the arcs (i, j) that compose the sub path $(m, n) \in \mathbb{F}$. Hence, in a double track the sets used are $\mathbb{N}$ and $\mathbb{A}$, and for a single track the sets are $\mathbb{G}$, $\mathbb{F}$ and $\mathbb{D}$. Therefore, the objective function that maximizes the total number of trains traversing the railway system is as follows:

$$\max_x \sum_{r \in \mathbb{C}} \sum_{k \in \mathbb{K}} x_r^k \tag{29}$$

Subject to

(30)

$$P^j_{i,j} \ = \ \frac{1}{T}\left[\sum_{k\in\mathbb{K}}(\alpha_{j,k}\ y^k_{i,j}\ +\widehat{y}^k_{i,j}\ \max_{(j,i)\in\mathbb{A}}\theta^k_{j,n})\right] \qquad \forall j\in\mathbb{N},\ \forall(i,j)\in\mathbb{A}$$

(31)

$$\sum_{(i,j)\in\mathbb{A}} P^j_{i,j}\ \leq 1 \qquad \forall j\in\mathbb{N}$$

(32)

$$P^j_{i,j} \ = \ \frac{1}{T}\left[\sum_{k\in\mathbb{K}}(\alpha_{j,k}\ y^k_{i,j}\ +\widehat{y}^k_{i,j}\ \max_{(j,n)\in\mathbb{F}}\beta^k_{j,n})\right] \qquad \forall j\in\mathbb{G},\ \forall(m,j)\in\mathbb{F},\ \forall(i,j)\in\mathbb{D}_{m,j}$$

(33)

$$\sum_{(m,j)\in\mathbb{F}}\ \sum_{(i,j)\in\mathbb{D}_{m,j}} P^j_{i,j}\ \leq 1 \qquad \forall j\in\mathbb{G}$$

(34)

$$\widehat{y}^k_{i,j} = (y^k_{i,j}+\widehat{y}^k_{i,j})\left[\frac{P^j_{i,j}\ \pi^j_{i,j}}{(1\ -\ \pi^j_{i,j})}\right] \qquad \forall k\in\mathbb{K},\forall j\in\mathbb{N},\ \forall(i,j)\in\mathbb{A}$$

(35)

$$y^k_{i,j}+\widehat{y}^k_{i,j}\ \ =\ \sum_{r\in\Omega_{(i,j)}} x^k_r \qquad \forall k\in\mathbb{K},\ \forall(i,j)\in\mathbb{A}$$

(36)

$$\sum_{k\in\mathbb{K}}(\theta^k_{i,j}+\alpha^k_j)\ (y^k_{i,j}+\widehat{y}^k_{i,j})\ \leq\ T \qquad \forall(i,j)\in\mathbb{A}$$

(37)

$$\sum_{k\in\mathbb{K}}\ \sum_{(i,j)\in\mathbb{D}_{m,n}}((\theta^k_{i,j}+\alpha^k_j)\ (y^k_{i,j}+\widehat{y}^k_{i,j})+(\theta^k_{j,i}+\alpha^k_i)\ (y^k_{j,i}+\widehat{y}^k_{j,i}))\ \leq\ T \qquad \forall(m,n)\in\mathbb{F}$$

(38)

$$\widehat{x}^k_r\ =\ \sigma_r\ x^k_r \qquad \forall r\in\mathbb{C},\ \forall k\in\mathbb{K}$$

(39)

$$\widehat{y}^k_{i,j}\ \leq\ \sum_{r\in\Omega_{(i,j)}}(1\ -\ \sigma_r)\ x^k_r \qquad \forall(i,j)\in\mathbb{A},\ \forall k\in\mathbb{K}$$

(40)

$$\pi^j_{i,j}\ =\ \sum_{(i',j)\in\mathbb{A}} P^j_{(i',j)}\ -\ P^j_{i,j} \qquad \forall j\in\mathbb{N},\ \forall(i,j)\in\mathbb{A}$$

(41)

$$\sigma_r\ =\ \prod_{j\in\mathbb{N},(i,j)\in\mathbb{S}_r}\mu^j_{i,j} \qquad \forall j\in\mathbb{N},\ \forall(i,j)\in\mathbb{A}$$

(42)

$$\mu^j_{i,j}\ =\ \left[1\ -\ \frac{P^j_{i,j}\ \pi^j_{i,j}}{(1\ -\ \pi^j_{i,j})}\right] \qquad \forall j\in\mathbb{N},\ \forall(i,j)\in\mathbb{A}$$

(43)

$$\beta^k_{m,n}\ =\ \sum_{(i,j)\in\mathbb{D}_{m,n}}\theta^k_{i,j} \qquad \forall k\in\mathbb{K},\forall(m,n)\in\mathbb{F}$$

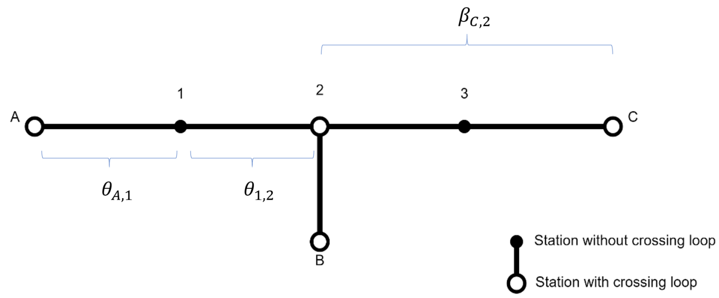

FIG. 1. Traveling time vs. headway time

The approach takes into account nine different sets of decision variable: x_r^k corresponds to the number of trains traveling through path r by the type of train k, including the second variable $\widehat{x}_r^k$ associated with the trains not delayed, $\widehat{y}_{i,j}^k$ related to the irregular train on section (i,j) , and $y_{i,j}^k$ for the regular train on section (i,j). The other sets of variable are described together with the explanation of the constraints.

Eq. (30) and Eq. (31) represent the conflict probability in a specific node belonging to a network with double track, and it is an extension of the Eq. (25). Thus, the time that the node will be occupied is the sum of the dwell time multiplied by the number of regular trains (i.e. trains without delay) plus the time that the trains must be waiting in the node, if the next section is being used by a train that is going in the same direction (headway time) multiplied by the number of irregular trains (trains with delay). Furthermore, the sum of all arcs that enter into the node j must be less than or equal to one.

Let us note that to determine the corresponding headway times, it is necessary to find the maximum of β. This situation happens when the node presents a bifurcation for the next section, as it is possible to notice in Fig. 1.

Being rigorous, in this type of issue it would not be necessary to use a maximum of respective time for each section belonging to the bifurcation. It would be more appropriate to use a probability of occurrence to each section or a stochastic approximation as suggested in [15]. On the other hand, [1] uses the Eq. (6) (non-linear) based on the definition given by [7], to estimate the headway time. However, that kind of estimation will not be considered in this study.

For the constraints associated for a network with single track, Eq. (32) and Eq. (33) follow the same structure. The only difference is the set where they apply (see the introduction of this section). Thereby, in networks with single tracks, only the exchange of trains is possible at nodes or stations with crossing loops. Therefore, the headway time, is now the traveling time of the trains crossing a section bounded by crossing loops, and this section could include some simple stations (without crossing loop). To clarify the concepts, the Fig 1. shows the section $(C,2)$ that includes the station 3 which does not have crossing loop. Moreover, the headway time for a train that is going to cross the section $(2,C)$, will be $\beta_{C,2}$, i.e. the time

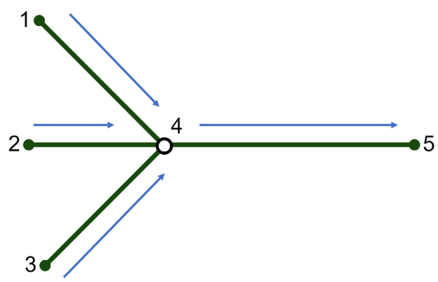

FIG. 2. Illustration of the assumption taken by (34)

it takes a train to cross the section $(C, 2)$. Thus, the nodes j are the ones which have crossing loops (i.e. $j \in \mathbb{G}$), and the arcs (i, j) are those belonging to a sub path bounded by two crossing loops (i.e. $(i, j) \in \mathbb{D}$).

Eq. (34) is almost equal to Eq. (26). The only difference lies in that (34) is for a section (i, j) and (26) is for a path r, but the concept behind the equations does not change and continues representing that the irregulars trains are a percentage of the total number of trains. However, (34) makes a strong assumption that must be explained in detail. To improve the understanding, Fig. 2 shows an hypothetical situation, where three sections enter at a node to continue in one section $(4, 5)$. Thus, the traffic conflict represented in the Eq. (34) and illustrated in the figure, assumes that the conflict in station 4 for a train that comes from the station 1 will not be with previous trains that have come from the same station 1, rather it will be with trains that come from section 2 or 3. Thereby, for the hypothetical situation represented in Fig. 2:

* $\pi^4_{1,4}$ will represent the probability of conflict from trains that come from station 2 or 3.
* $1 - \pi^4_{1,4}$ is the probability that the section $(4, 5)$ is not occupied.
* $P^4_{1,4}$ is the probability that the node j is occupied, when the train comes from station 1.

Constraint (35) defines that the sum between irregular and regular trains for a particular section (i, j) must be equal to the total number of trains that follow the path r crossing section (i, j).

The inequality (36) indicates that the trains, whether they are regular, irregular or both, must complete the section (i, j) in less time than the evaluation period, considering the traveling and dwell time (time that the train must be stopped at a station or node, because of blocking of the section (i, j) by an outgoing section (i.e. the section $(4, 5)$ in Fig 2)). Let us note that the constraint applies for double track section as the traveling time is considered in one direction. If it is the case of a single track, the Eq. (37) must incorporated, because it takes into account both directions within the traveling time. However, in this situation the inequality must add all the arcs (i, j) belonging to a path r bounded by two crossing loops, i.e. $(i, j) \in \mathbb{D} : \mathbb{D} \subseteq \mathbb{F}$.

The equalities (38), (39), (41) and (42) are related to each other. First, $\mu^j_{i,j}$ is the probability that a node j belonging to a section (i,j) is not occupied, and σ_r is the probability that train on path r is not delayed. Thus, the Eq. (38) states that the number of trains of type k, without delay that cross a path r must be equal to the probability of finding a free path r multiplied by the total number of trains crossing that path for all types of trains. Likewise, the Eq (39) indicates that the number of trains with delay crossing section (i,j) is equal to the probability that the path r is busy multiplied by the total number of trains that are crossing section (i,j).

Eq. (43) is related only for the cases where the network presents a single track configuration, because it incorporates the traveling time between two bounded points, that corresponds to stations with passing loop. Finally, to make the model comparable with the next one, and to include the proportion of trains circulating on the network, Eq. (1) and (2) presented at the beginning of this report, must be added.

Appendix A, contains a summary of the equations corresponding to the *Single-track non-linear problem 1* and *Double-track non-linear problem 1.*

1.1. A previous reformulation. The formulation presented previously is a non-linear programming problem because of constraints (30) and (34) being non-linear. Specifically, the estimation of the conflicting probabilities makes the other constraints become non-linear. Thus, the approach for solving the problem is the same used by [2], consisting on freezing the probability $P^k_{i,j}$, thus producing that the variables μ, β, σ and π, become parameters. However, it is possible to observe that Eq (31) includes a maximum value of traveling time. To avoid this issue in a linear problem, new parameters must be added.

$$\widehat{\theta}^k_{i,j} = \max_{(j,n)\in\mathbb{A}} \{\theta^k_{j,n}\} \qquad \forall j \in \mathbb{N},\ \forall (i,j) \in \mathbb{A}, \forall k \in \mathbb{K}, n \neq i \tag{44}$$

$$\widehat{\beta}^k_{i,j} = \max_{(j,n)\in\mathbb{F}} \{\beta^k_{j,n}\} \qquad \forall j \in \mathbb{G},\ \forall (i,j) \in \mathbb{D}_{m,j}, \forall (m,j) \in \mathbb{F}, \forall k \in \mathbb{K} \tag{45}$$

Both parameters will depend on whether the network uses single or double track. Hence, in the case of single track, the parameter $\widehat{\beta}^k_{i,j}$, must be included in the Eq (32), and in the double track case, the parameter $\widehat{\theta}^k_{i,j}$ for the Eq. (30). Thereby, keeping the objective function, the new formulation for the new linear problem must consider the following modifications:

$$\sum_{k\in\mathbb{K}} \sum_{(i,j)\in\mathbb{A}} (\alpha_{j,k}\ y^k_{i,j} + \widehat{y}^k_{i,j}\ \widehat{\theta}^k_{i,j}) \leq T \qquad \forall j \in \mathbb{N} \tag{46}$$

$$\sum_{k\in\mathbb{K}} \sum_{(m,n)\in\mathbb{F}} \sum_{(i,j)\in\mathbb{D}_{m,n}} (\alpha_{j,k}\ y^k_{i,j} + \widehat{y}^k_{i,j}\ \widehat{\beta}^k_{i,j}) \leq T \qquad \forall j \in \mathbb{G} \tag{47}$$

(46) replaces the set of equations (30) and (31) corresponding to the double track, and (47) is equivalent to (32) and (33) for single tracks.

The Eq. (34), (35), (36), (37), (38) and (39) are kept unchanged, and the variables π, σ and μ, are now parameters given by (40), (41) and (42) respectively, and also keep their structure. Nevertheless, for the variable $P^j_{i,j}$, now also a parameter, the following modification must be considered.

$$(48) \quad P^j_{i,j} = \frac{1}{T}\left[\sum_{k\in\mathbb{K}}(\alpha_{j,k}\ y^k_{i,j}\ + \widehat{y}^k_{i,j}\ \widehat{\theta}^k_{i,j})\right] \qquad \forall j \in \mathbb{N},\ \forall (i,j)\in\mathbb{A}$$

$$(49) \quad P^j_{i,j} = \frac{1}{T}\left[\sum_{k\in\mathbb{K}}(\alpha_{j,k}\ y^k_{i,j}\ + \widehat{y}^k_{i,j}\ \widehat{\beta}^k_{i,j})\right] \quad \forall j \in \mathbb{G},\ \forall (m,j)\in\mathbb{F}, \forall (i,j)\in\mathbb{D}_{m,j}$$

It is necessary to bear in mind that all the variations associated with the traveling times, will always come in couples, because, it will depend on whether the railway possesses double or single track, and of course, in the probable case that the network includes both types, both equations must be considered at the same time.

Appendix A, contents the summary formulation for *Single-track linear problem 1* and *Double-track linear problem 1*

1.2. The fixed-point heuristic method. The fixed-point method is the iterative process used to solve the linearization of approach 1. Thus, to start it is necessary to define initial values of $P^j_{i,j}$, that can be anywhere between $(0,1)$, and that will allow prior calculation of values π, σ and μ to solve the initial problem, as all of them depend on the values taken by $P^j_{i,j}$.

Once the initial linear problem is solved for the given values, the parameters must be recomputed as a function of the solution obtained, and the initial values of x^k_r, associated with the maximum number of trains allowed in the network, must be saved in auxiliary parameters that we call $X^{(0)}$, where the zero indicates that it is the initial iteration

To continue, the problem must be solved again, taking into account the new values of the parameters obtained in the previous step, and the results saved in the auxiliary parameter $\widehat{X}^{(u)}$. However, the values for x^k_r, saved in $\widehat{X}^{(u)}$, are not now used to continue the algorithm directly. Thus, before the next step, the new values for x^k_r must be estimated by the following equation:

$$X^{(u)} = X^{(u-1)} + \xi_{(u)}(\widehat{X}^{(u)} - X^{(u-1)})$$

The previous expression is widely solved with Krasnoselskii-Mann iterative schemes (Krasnoselskii (1954), Mann (1953)). An enhancement was proposed by Ishikawa (Ishikawa(1974)) that achieved a better rate of convergence and it consisted of a double step which required two evaluations of the point-to-set mapping at each iteration. These types of iterative schemes were initially developed by Blum (1954) and Robins and Monro (1951) and are also used for fixed-points of single-valued maps and for solving stochastic equations. In transportation modeling, they are widely known under the name Methods of Successive Averages (MSA). These iterative methods can be summarized as:

$$X^{u+1} = X^u + \alpha_u\{\widehat{X}(X^u + \beta_u(\widehat{X}(X^u) - X^u)) - X^u\}$$

Where the point-to-set map $\widehat{X}$ in our case would be defined by the solutions of the parametrized linear program. The convergence of these iterative methods to a fixed-point has been proved under different hypotheses for the sequences α_u y β_u. The convergence of these iterative methods to a fixed-point has been proved under different hypotheses for the sequences α_u y β_u and some non-expansiveness properties of the map $\widehat{X}$ (see, for instance, Dunn (1978) and Maiti and Ghosh (1989)). For the case $\beta = 0$ the Krasnoselskii-Mann (or MSA) iterative scheme is obtained. In general, the Ishikawa iterative scheme has a better convergence rate than Mann's method in terms of the number of iterations. However, Ishikawa's method requires two evaluations of the point-to-set map per iteration, whereas Mann's method requires only one. For this case is considered $\beta = 0$ and $\alpha_u = \xi_{(u)}$ where $\xi_{(u)} = \frac{1}{1+u}$. The overall explanation of the iterative process described above, can be seen in [4].

$X^{(u)}$ represents the new values that must be taken in the iterative process. Thereby, the method will continue iterating until the relative error between consecutive solutions is less or equal than ε, where ε is a small non negative number.

Finally, the process described above, can be summarized as follows:

(1) Initial:
   (a) For an initial value of $P_{i,j}^j = 0.05$ it is possible to obtain the values for $\pi_{i,j}^j$, $\mu_{i,j}^j$, σ_r.
   (b) Solve [Capacity] linear problem
   (c) Compute the new values for $P_{i,j}^j$, $\pi_{i,j}^j$, $\mu_{i,j}^j$, σ_r.
   (d) $u = 1$, $X^{(0)} = x_r^{k(0)}$.
(2) While $(\varphi \geq \varepsilon)$ do:
   (a) Solve [Capacity] linear problem
   (b) $\widehat{X}^{(u)} = x_r^{k(u)}$
   (c) Compute $X^{(u)} = X^{(u-1)} + \xi_{(u)}(\widehat{X}^{(u)} - X^{(u-1)})$
   (d) $\varphi = \frac{\|X^{(u)} - X^{(u-1)}\|}{\|X^{(u)}\|}$
   (e) Update $X^{(u-1)} = x_r^{k(u)}, u = u + 1$.
   (f) Compute the new values for $P_{i,j}^j$, $\pi_{i,j}^j$, $\mu_{i,j}^j$, σ_r.
   (g) End While

2. A fixed-point heuristic method for solving non-linear approach 2

As it was mentioned in the previous chapter, this second approach is based on the work done by [1] and contains less constraints than the previous approach, because of the conflict in the nodes being tackled in a different way. Specifically, this approach changes the concept of conflict probability used in the first model and introduce the concept of rolling stock suggested by [1], that corresponds to the sequence of trains that are traveling in one direction or in another, associate to the Eq. (1) and (2). Furthermore, considering that this model is an extension of [1], it will also provide a range of solutions regarding the total number of trains allowed on the network, being the maximum of trains as consequence of the optimal programming of the rolling stock, and the minimum of the worst combination of trains.

The fact that the approach 2 has less constraints than model 1, it will influence in the results when these approaches will be compared, i.e. in the outcomes, the maximum number of trains provided by the formulation 2, will be higher than the first model, because the second model is less restrictive.

The set of variables is also smaller than in previous approach because, in this case the number of trains delayed is not considered as a variable of the model. Hence, two sets of important variables associated to the number of trains of a given type k traveling through a particular path are used with one at corridor level (x_r^k), and the same variable but at section level ($y_{i,j}^k$).

The objective function (29) continues to be the same for this approach, however, some constraints present small variations:

Eq. (50) has a similar structure to (35), and its meaning is strictly the same, i.e. the total trains of type k, traveling on a section (i, j) must be obtained considering the flows of train of type k and all path r that use section (i, j). (51) is a new family of constraints and defines the variables $Y_{i,j}$, total number of trains traveling in the section (i, j), considering all types of trains.

Eq. (53) is equal to Eq. (36) with the only difference that in (53) the variable takes the total of trains and does not make any difference between delayed and non-delayed trains. The same case happens between (52) and (37), only that these equations are for single tracks, and (53) is for double tracks.

The constraints that must be included in the model in order to estimate the lower bound of the total capacity of the railway system are (54) and (55) depending on whether the network consists of single or double tracks respectively. Both equations include the dwell times and the headway time in their formulation. However, despite that the left parts of the constraints are equal, the right side is different, because the headway time concept used is not the same. Let us recall that, for a single track the time that must be taken into account will be the traveling time between two stations with crossing loop, and in the double track, this *time* will only be the traveling time between two stations, no matter if it has crossing loop or not. Furthermore, to be solved, both equations need a prior computation of the maximum headway time over a node, or the minimum number of trains traveling in a section in opposite direction (fleet). Thus, through (54) the non-linearity arises.

(50)
$$y^k_{i,j} = \sum_{r\in\Omega_{(i,j)}} x^k_r \qquad \forall k \in \mathbb{K}, \forall(i,j)\in\mathbb{A}$$
(51)
$$Y_{i,j} = \sum_{k\in\mathbb{A}} y^k_{i,j} \qquad \forall(i,j)\in\mathbb{A}$$
(52)
$$\sum_{k\in\mathbb{K}} \sum_{(i,j)\in\mathbb{D}_{m,n}} ((\theta^k_{i,j}+\alpha^k_j)\ y^k_{i,j} + (\theta^k_{j,i}+\alpha^k_i)\ y^k_{j,i}) \ \leq\ T \qquad \forall(m,n)\in\mathbb{F}$$
(53)
$$\sum_{k\in\mathbb{K}} (\theta^k_{i,j}+\alpha^k_j)\ y^k_{i,j} \leq\ T \qquad \forall(i,j)\in\mathbb{A}$$
(54)
$$\sum_{k\in\mathbb{K}} \left[\alpha^k_j y^k_{i,j} + \left\{\max_{(j,n)\in\mathbb{F}} \beta^k_{j,n}\right\} \min(Y_{i,j}, Y_{j,i})\right] \leq T \qquad \forall j\in\mathbb{G}, \forall(m,j)\in\mathbb{F}, \forall(i,j)\in\mathbb{D}_{m,j}$$
(55)
$$\sum_{k\in\mathbb{K}} \left[\alpha^k_j y^k_{i,j} + \left\{\max_{(j,n)\in\mathbb{A}} \theta^k_{j,n}\right\} y^k_{i,j}\right] \leq T \qquad \forall j\in\mathbb{N},\ \forall(i,j)\in\mathbb{A}, n\neq i$$
(56)
$$\sum_{k\in\mathbb{K}} (\alpha^k_j y^k_{i,j}) \leq T \qquad \forall k\in\mathbb{K}, \forall(m,j)\in\mathbb{F}, \forall(i,j)\in\mathbb{D}_{m,j}$$
(57)
$$\sum_{k\in\mathbb{K}} (\alpha^k_j y^k_{i,j}) \leq T \qquad \forall(i,j)\in\mathbb{A}$$

Inequalities (56) and (57) are associated with the upper bound supported by the network for a single and double track, being the latter, the reason why the only difference between them is the set where the inequalities are applied. It is possible to observe that these constraints only include dwell times, since in order to obtain the maximum number of trains on the railway system, the optional sequencing of them, must be every one in one direction, and then all in the opposite direction. In this way, trains will not be delayed at nodes, and no enforced headway will take place at node. (See in Appendix A *Single-track non-linear problem 2* and *Double-track non-linear problem 2*)

2.1. A previous reformulation. To linearize the non-linear problem, for the single track case, some continuous and binary variables must be incorporated along with their respective constraints. Therefore, the inequality (55) that obtains a lower bound for a double track system, will be replaced by (46). And the Eq. (54) associated for a single track network, will be changed by the following constraint:

$$\sum_{k \in \mathbb{K}} \left[\alpha_j^k \ y_{i,j}^k \ + \ \widehat{\beta}_{i,j}^k \ Q_{i,j} \right] \ \leq \ T \qquad \forall k \in \mathbb{K}, \forall (m,j) \in \mathbb{F}, \forall (i,j) \in \mathbb{D}_{m,j} \tag{58}$$

Eq. (58) has equal structure to (47), however, in this equation, $\widehat{\beta}_{i,j}^k$ (headway time applied for a single track) is multiplied by a new continuous *positive* variable $Q_{i,j}$. The variable is related with the minimum sequence of trains allowed in a section (i,j), and (58) becomes a linear constraint. On the other hand, the use of this new variable, requires new constraints that allow the relation with the other variables of the problem.

Eq. (59) and (60) are the upper bound for $Q_{i,j}$. Hence, the maximum value that $Q_{i,j}$ may take will be the maximum number of trains traveling in the section (i,j), this is $Y_{i,j}$ or $Y_{j,i}$. (61) forces both binary variables to sum up to one, where $d1_{i,j}$ is associated when $Y_{i,j}$ is the minimum and $d2_{i,j}$ when $Y_{j,i}$ is the minimum.

$$Q_{i,j} \ \leq \ Y_{i,j} \qquad \forall k \in \mathbb{K}, \forall (i,j) \in \mathbb{A} \tag{59}$$

$$Q_{i,j} \ \leq \ Y_{j,i} \qquad \forall k \in \mathbb{K}, \forall (i,j) \in \mathbb{A} \tag{60}$$

$$d1_{i,j} \ + \ d2_{i,j} \ = \ 1 \qquad \forall k \in \mathbb{K}, \forall (i,j) \in \mathbb{A} \tag{61}$$

$$Q_{i,j} \ \geq \ Y_{i,j} \ - \ M(\ 1 - d1_{i,j}) \qquad \forall k \in \mathbb{K}, \forall (i,j) \in \mathbb{A} \tag{62}$$

$$Q_{i,j} \ \geq \ Y_{j,i} \ - \ M(\ 1 - d2_{i,j}) \qquad \forall k \in \mathbb{K}, \forall (i,j) \in \mathbb{A} \tag{63}$$

Finally, Eq. (62) indicates that if $Y_{i,j}$ is the minimum between $(Y_{i,j}, Y_{j,i})$, then $Q_{i,j} \geq Y_{i,j}$ and $Y_{i,j} \geq Q_{i,j}$, hence $Q_{i,j}$ takes the value of $Y_{i,j}$. And for (63) applied for the same situation ($Y_{i,j}$ minimum), $Q_{i,j} \geq Y_{j,i} \ - \ M$, where M is a big number, and as $Q_{i,j}$ must be positive, (63) is a superfluous constraint. And the same reasoning can be applied when $Y_{j,i}$ is the minimum value. (The summary formulation can be seen in Appendix A in *Single-track linear problem 2* and *Double-track linear problem 2*)

2.2. The fixed-point heuristic method. The non-linearity emerged by the approach 2, has already been eliminated in the previous section when the variable $Q_{i,j}$ was introduced. However, if in future extensions, the way to estimate the headway time is different to the one proposed in this master thesis, and the equation to estimate it is as suggested by [1], i.e. using the Eq. (6), the fix-point method will also be a good methodology to tackle the non-linearity raised by this constraint (6).

Therefore, for the single track and with a estimation of $\beta_{i,j}^k$ under the Eq. (6), the steps for the algorithms will be exactly the same ones that were used in the first approach, the only difference lying in the estimation of the initial point. Thus, to find the first solution, the linear problem that must be solved considers the Eq. (1), (2), (50) and (51) plus the follow constraint

$$\sum_{r \in \Omega_{i,j}} \sum_{k \in \mathbb{K}} \theta_{i,j}^k x_r^k \ \leq \ T \qquad \forall (i,j) \in \mathbb{A} \tag{64}$$

Thus, the linear problem (i.e. step (a) of the algorithm) used to find the initial solution represents the simplest formulation to estimate the railway capacity, since that considers only heterogeneous composition of train flows.

Finally, the iterative process, described with detail in the solving methodology for the linear approach 1, can be summarized as follows:

(1) Initial:
   (a) Solve [Abs Network Capacity] linear problem using Eq. (1), (2), (50), (51) and (64)
   (b) Compute the values for $\beta^k_{i,j} \quad \forall (i,j) \in \mathbb{A}$
   (c) $u = 1$, $X^{(0)} = x_r^{k(0)}$.
(2) While $(\varphi \geq \varepsilon)$ do:
   (a) Solve [Abs Network Capacity] for the second linear approach
   (b) $\widehat{X}^{(u)} = x_r^{k(u)}$
   (c) Compute $X^{(u)} = X^{(u-1)} + \xi_{(u)}(\widehat{X}^{(u)} - X^{(u-1)})$
   (d) $\varphi = \frac{\|X^{(u)} - X^{(u-1)}\|}{\|X^{(u)}\|}$
   (e) Update $X^{(u-1)} = x_r^{k(u)}, u = u + 1$.
   (f) Compute the new values for $\beta^k_{i,j}$ $\forall (i,j) \in \mathbb{A}$
   (g) End While

3. Extension of the models

3.1. Extension A: Minimum traffic. Both approaches previously commented have an objective function (29) that maximizes the total number of trains on the railway system. However, for the formulations presented until now, the algorithm will tend to discard all long-distance paths and will select mostly, the shortest path for each origin / destination pair. Therefore, using a formulation without a minimum number of trains for each line could generate a distortion of the estimate a capacity obtained by the algorithms when applied to realistic networks on which a given number of lines is already operating in order to satisfy some specific demand.

To avoid this issue, and with the aim of comparing the approaches in a real situation, the following constraint must be added for each one. Nevertheless, it is necessary to stress that the Eq (65) is an oversimplification for the minimum demand satisfaction requirement on the network.

$$\sum_{k \in \mathbb{K}} x_r^k \geq LB_r \qquad \forall r \in \mathbb{C} \tag{65}$$

The parameter LB_r represents the minimum amount of trains that the network requires, for each path. A finer extension would be including additional constraints that take into account the demand requirements for each type of train and for each type of service.

3.2. Extension B: An alternative formulation for approach 2. Let us recall that the main difference between both models presented is that the second approach does not distinguish the delayed trains and the outcome is a range of maximum capacity. i.e. a lower and upper bound. However, the incorporation of some continue and binary variables related with the travel time, and with their respective constraints, allow finding a value between this optimal rank, for the case of double and single track.

Additional variables

Two sets of continuous variables must be introduced. The first one, τ_r^k which is the departure time between two consecutive trains of type k, following the path r. And the second t_r^k which is the traveling time on path r, by train of type k. Furthermore, a binary variable δ_r^k must be included that indicates whether the train k, crosses the path r.

Additional parameters

A new parameter $\overline{\theta}_r^k$ is required, that represents the travel time in the first section belonging to the path r, for each train of type k. And another parameter h_r^k, that will represent the arriving time between two consecutive trains of type k, following the path r.

When *Double-track* case is considered, the following equations must be added, to replace the Eq. (55) and (57).

$$\tau_r^1 + \delta_r^1 \rho_r^1 + h_r^1 x_r^1 <= t_r^1 \tag{66}$$

$$\tau_r^k + \delta_r^k \rho_r^k + h_r^k x_r^k <= t_r^k \tag{67}$$

$$\tau_r^k \geq \tau_r^{k-1} + h_r^{k-1} x_r^{k-1} + \overline{\theta}_r^k \delta_r^k \tag{68}$$

$$\tau_r^k \leq T \tag{69}$$

$$\delta_r^k \leq x_r^k \leq M_r \delta_r^k, \qquad \delta_r^k \in \{0,1\} \tag{70}$$

The approach two must be modified, including the set of equations (66) (70) and remove (55) and (57) associated with lower and upper bound respectively. Thus, the solution obtained using this new formulation will be a value between the rank of optimal capacity given by the original model 2. Specifically, (66) represents the travel time for the first train on the network, where is added the starting time τ_r^k, plus the travel time in crossing the path r ρ_r^k multiplied by the binary variable δ_r^k if the train effectively is using path r, and the headway time h_r^k (which is the the arriving time for two consecutive trains), which must be less than the time variable t_r^k. Likewise, the Eq (67) represents the time for every other train that is crossing the path r belonging to the system. On the other hand, (68) indicates that the departure time for a train of type k that is going to start, must be greater than the sum of the starting time and headway time for the previous train, plus the travel time of the first section $\overline{\theta}_r^k$ of the path for the train that is going to start the travel.

(69) and (70) represent in first instance, that the variable t_r^k must be less or equal to the period of time T, and secondly, the relation between the binary variable with the variable x_r^k.

For the *Single-track* case must be taken into account the Eq.(66), (67), (68), (70) and some additional constraints are included:

$$\tau_{r'}^1 \geq t_r^k \tag{71}$$

$$\tau_{r'}^1 + \delta_{r'}^1 \rho_{r'}^1 + h_{r'}^1 x_{r'}^1 <= t_{r'}^1 \tag{72}$$

$$\tau_{r'}^k + \delta_{r'}^k \rho_{r'}^k + h_{r'}^k x_{r'}^k <= t_{r'}^k \tag{73}$$

$$\tau_{r'}^k \geq \tau_{r'}^{k-1} + h_{r'}^{k-1} x_{r'}^{k-1} + \overline{\theta}_{r'}^k \delta_{r'}^k \tag{74}$$

$$\tau_{r'}^k \leq T \tag{75}$$

$$\delta_{r'}^k \leq x_{r'}^k \leq M_{r'} \delta_{r'}^k, \qquad \delta_{r'}^k \in \{0,1\} \tag{76}$$

The set of equations from (72) to (77) are exactly equal to previous sets of equations, but for a train that follows the path r', where if r is the path which unites the pair origin / destination (p,q), then r' unites the pair (q,p). Let us bear in mind that, travel in both directions must be considered on a single track. Thus, the Eq (71) ensures that the trains that start the travel from q to p, must do it after the train that follows the path r has already finished.

Finally, the additional parameters have the following compute equations:

$$h_r^k = \max_{(i,j)\in S_r} \{\theta_{i,j}^k\} \qquad \forall r \in \mathbb{C} \tag{77}$$

$$\overline{\theta}_r^k = \theta_{i,j}^k \qquad \forall (p,q) \in \mathbb{R},\ \forall k \in \mathbb{K},\ \forall r \in \xi_{p,q},\ \forall (i,j) \in \mathbb{S}_r : i = p \tag{78}$$

4. Automatic generation of parameters and sets

Part of the sets used to resolve the formulations set out in this master thesis require a special attention, because some of them are not easy to create and could lead to consistency errors if a generation algorithm is not used. Therefore, to deal with this issue, some sets will be generated automatically based on other sets that define the network. Thus Table 1. shows which sets will be generated through the algorithm

To create the sets belonging to the column at the right side, it will be necessary solve a shortest path problem, to find all the possible paths that join an origin / destination pair. This process must be developed through a generation algorithm because each one of these sets is composed by several elements, and if one of them is not found, at the moment of solving the problem, the result will be unfeasible or not bounded .

By hand	By Algorithm
$\mathbb{N}$	ξ
$\mathbb{R}$	$\mathbb{S}$
$\mathbb{A}$	Ω
$\mathbb{G}$	$\mathbb{D}$
$\mathbb{F}$	$\mathbb{C}$

TABLE 1. sets

For instance, to Ω set, that are all path r that cross a section (i, j), would be a great work to establish all paths that cross it and furthermore it would not be easy to consider a path in a big network. Thus, to avoid this issue, first it is necessary, through the shortest algorithm without sub-circuits, to find all possible path r for each origin / destination pairs, and then find the paths that cross a section (i, j) for each one of them belonging to $\mathbb{A}$

Therefore, there are two problems that must be solved: the first one, is a simple optimization problem with just a balanced constraint, and the second one, to create the amount of possible paths, it should include constraints that avoid sub-circuits.

$$\min_{Link} \sum_{(i,j)\in\mathbb{A}} Cost_{i,j} Link_{i,j} \tag{79}$$

$$\sum_{(i,j)\in\mathbb{A}} Link_{i,j} - \sum_{(j,i)\in\mathbb{A}} Link_{j,i} = \begin{cases} 1 & if\ i = p \\ 0 & if\ (i,j) \neq (p,q) \\ -1 & if\ j = q \end{cases} \tag{80}$$

$$\sum_{(i,j)\in\mathbb{A}} Link_{i,j} Link^0_{i,j} \leq \sum_{(i,j)\in\mathbb{A}} |Link^0_{i,j}| - 1 \tag{81}$$

$$\sum_{(i,j)\in\mathbb{A}} Cost_{i,j} Link_{i,j} \geq Cost_{i,j} Link^0_{i,j} + \min_{(i,j) in \mathbb{A}} Cost_{i,j} \tag{82}$$

In the problem above, $Link_{i,j}$ is the binary variable which represents a section (i, j), the parameter $Cost_{i,j}$ is the cost of each section and their values are generated randomly. $Link^0_{i,j}$ is a vector of ones and consequently another parameter that is necessary in constraint to avoid sub-circuits, and (p, q) represent the origin / destination points.

Finally, all the sets are generated using the complete algorithm and only the set $\mathbb{D}$ uses the Eq. (79) and (80)

Chapter 5
Computational results

1. Medium sized railway network

To apply the approaches presented, this thesis considered the Line 1, 2, 3 and 7 of Rodalies Network of Catalunya, shown in Fig. 1, with 124,4 km of tracks. The railway system includes elements of the three parameter categories that influence over the capacity. Specifically, it is composed by 8 points of origin / destination, 29 stations of which 16 can be considered without crossing loop, and three nodes. Furthermore, it has 6 types of trains, among them, 450, 447, Civia, 470, 449 and 448 to offer regional service and in Barcelona down town with a maximum speed of 120, 140 and 160 Km/h depending on the series.

Line 1 that is composed of 47.7 km of tracks, has been taken from Molins de Rei (B) to Mataró (E). For Line 2 has been considered only from Vilanova i la Geltrú (A) to Estació de França (C) with 48 Km. Line 3 has been taken into account from L'Hospitalet (16) to Granollers (F) with 36.5 km of tracks, and finally to Line 7, from Sant Andreu (24) to Cerdanyola Universitat (D) with 13,1 km. (Configuration of the network in Tables 15-16)

The network as it is considered, i.e. with stations without crossing loops, allows replacing the signals by these stations due to that the train must be stopped for a given time and does not allow the train flows in the opposite direction because it does not have an additional track. Hence, in a hypothetical situation, all the 16 simple stations can be considered as signals.

For the case study, only two out of six types of trains have been taken into account since the only relevant parameter is speed. Hence, the approaches will be tested for trains with average speed of 100 and 80 km/h for a period of 18 hours and including the dwell times given by standard scheduling. Furthermore, as the formulations have been designed to be applied in large-scale networks, all possible paths that join origin/destination pairs will be considered.

The models have been evaluated assuming two possible scenarios. In the first one, the network is composed only by double track segments, and in the second, the railway system has only single tracks in all segments. A mixed scenario has not

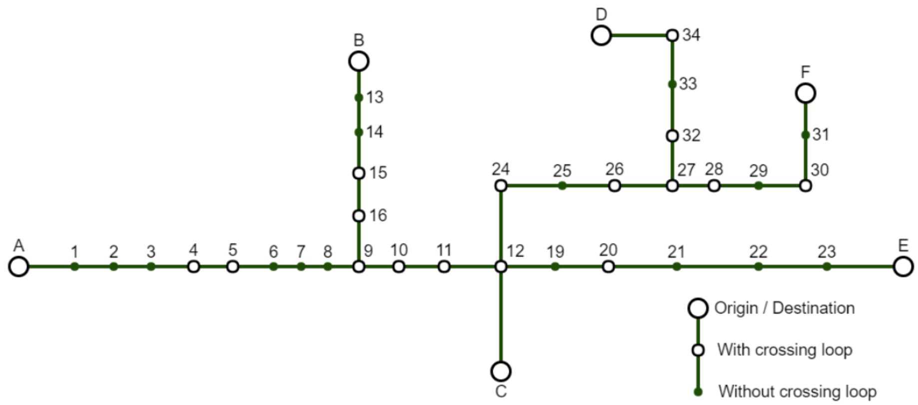

FIG. 1. Traveling time vs. headway time

been considered because the objective is to observe the percentage that influences the double and single-track factor in a network.

Finally, to facilitate the understanding of the results, all the tables discussed below are attached in Appendix B.

2. Double track network

2.1. General results. Table 1 indicates that traffic exists in 20 out of 48 possible paths, without considering a minimum of trains in each one of them and a given rolling stock and dwell time. Specifically, each column of the table presents the traffic distribution for each possible path obtained through the different formulations.

In Table 1 it is possible to observe that the differences between the results are related to linear and non-linear problems. For the first approach, they are very similar in terms of the total flows, but different with their traffic distribution. Thus, the linear approach 1 estimates the capacity of the network in 1.969,85 trains while the non-linear model 1 indicates 1.983,99 trains, giving a 0,72% of difference. On the other hand, the formulation 2 provides an optimal rank between 1.314,08 and 2.181,53 trains on the network for 18 hours. Thereby, the result of the first approach is within of the optimal rank provided by the model 2, and being precise a 9,05% below of the upper bound (2.181,53 trains)

Furthermore, the paths which have the highest train flows are those with the smallest length of track, this is the case of paths $(B, 16)$ and $(D, 24)$ that have 10 km and 13,1 km respectively, being the number of trains that cross these paths 372,78 and 338,79 trains in 18 hours (a train every 3 minutes). On the contrary, the longest track (A, C) has the lowest train flows with a total of 27,12 trains (one train every 40 minutes).

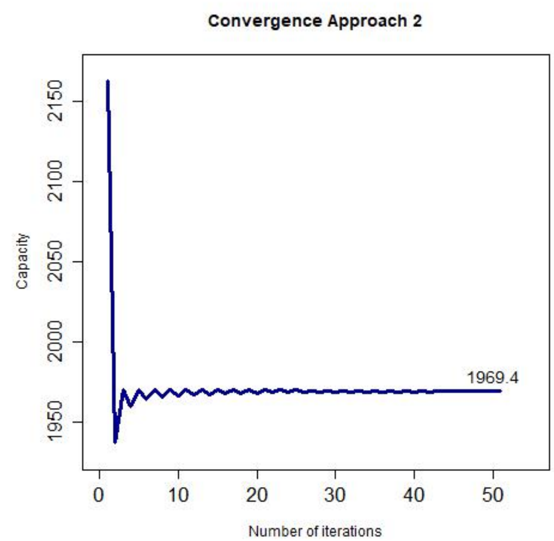

FIG. 2. Convergence of the iterative process

Table 2 shows the percentage of trains not delayed and delayed for each path. In general, all the paths that cross node 12, where 4 sections converge, has a high percentage of delayed, highlighting the paths $(E, 24)$ and $(24, E)$ that have the 100% of their trains with delay.

2.2. Rolling stock. The results related with Table 1 are under a hypothetical distribution of the mixture of trains associated to the parameters $\eta^k_{p,q}$ y $\nu^k_{p,q}$. However, in the case where these constraints are not taken into account, the capacity of the network would be increased by 35,6% (2.672,7 trains) and the lines used are reduced to only 10, marked by the shortest path of the network, see Table 5.

Moreover, within the 10 paths selected by the models, $(B, 16)$ and $(D, 24)$ continue being the paths with highest train flows with 554 and 480 trains in 18 hours respectively, that correspond to a train every two minutes. However, in these cases, the amount of trains produces an increase in the percentage of trains delayed for these lines particularly. Specifically, for the line $(B, 16)$ the percentage of trains delayed increase from 19% to 59% and the line $(D, 24)$ from 31% to 54%, with regard to the situation that considers the rolling stock parameters.

2.3. Minimum number of trains. As the algorithm does not give paths with more than three nodes, because the formulations tend to select the shortest path, the Table 6 shows the results of a hypothetical case when the models are forced to consider a minimum train flows for each path. Thus, in the case when the minimum number of trains on each path is greater than one, as constraint, the capacity is reduced by 7,86% from 1.969,85 trains to 1.808 regarding the Table 1, and the percentage of trains delayed increases from 40,2% (Table 2) to 45% (Table 6). However two aspects must be stressed:

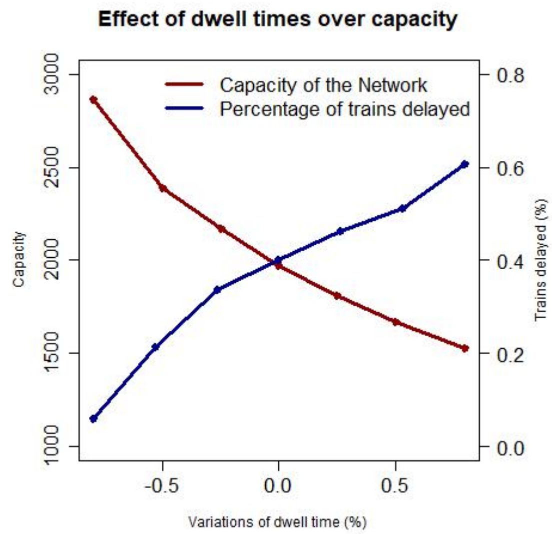

Fig. 3. Effect of Dwell times

First, although in the overall results the percentage of trains delayed only increased by 4,8%, the percentage of delays is high in most paths. In Table 6, it is possible to observe that 14 lines have the 100% of their trains delayed and 37 lines out over 48 possess more than 70% of the trains with delays. The reason for this disparity is explained because the paths with more traffic (i.e. a major amount of trains circulating) have less percentage of trains delayed. In other words, the formulations give priority to the lines with highest train flows.

Secondly, and in accordance with the results discussed in the previous paragraph, the shortest lines $(B, 16)$ and $(D, 24)$ continue having the highest number of trains circulating.

2.4. Dwell times. To determine how much an increase or decrease of the dwell times can affect the capacity or the percentage of trains with delay, the thesis has taken the common case studied until now for double track, and modified the dwell time, reducing it up to 80% regarding the base case, and then increasing it in the same proportion. Thus, by means of the linear approach 1, Fig 3 shows, with a red line, that the capacity increases to 45,4% (2.864,08) when the dwell times are 80% shorter than the base case, and in the case that the dwell times are an 80% longer than the base case, the capacity decreases from 1.969,85 to 1.526,3 trains in 18 hours (22,5%).

Through the blue line it is shown, that the percentage of trains delayed increases 51,34% with respect to the base case when the dwell times are greater in 80%, and in the case when the dwell times are an 80% shorter than the neutral case, the percentage of delays decreases until 5,9%, that means that the network almost does not have delays.

2.5. Headway time. It is a relevant parameter to be analyzed because it is directly related with the trains scheduling. As it was explained in the Chapter 3, if the scheduling is not efficient, there will be congestion in each station and the headway time will increase. On the other hand, if the railway system has a good timetable, the congestion at the stations will decrease and the headway can be reduced. Table 12 shows the results for the case study considering a double track. The second column indicates the maximum headway time allowed with the worst scheduling (the trains must wait in each station, because the next one is occupied), the third column is associated to the total dwell times and the fourth column shows the total travel time. Columns 5 and 6 report the minimum time for each path (i.e. without considering the headway time due to an efficient scheduling) and the maximum travel time considering the maximum headway time allows, respectively. Thereby, column 7 indicates by what percentage the headway time, or good scheduling, impacts the total travel times. Thus, the average impact over an efficient scheduling is 45,1%.

It is important to remember that the headway time it is not necessarily equal to the travel time, because in the nodes with more than one in coming section, the time considered is the maximum between the sections, as was explained in Fig 1 of chapter 4.

2.6. Convergence. Fig 2 shows the convergence of the iterative process for the second approach for the first 50 iterations. The first iteration provides 2.162,82 trains since it considers as initial point, a conflict probability of 10% for all stations, which is quite low. However, intermediately, in the second iteration, the algorithm gives 1.937,62 trains which is a near neighborhood to the optimal solution. Finally, after 50 iterations, the formulation indicates 1.969,4 as the maximum number of trains that can cross the railway system.

2.7. Using Extension B. Considering a period of time $T = 60$ minutes, Table 10 in appendix B, shows comparative results for flows in paths depending on the approach used. As was explained in chapter 4, the extension B is a modification of the formulation 2, such that, it deletes the constraints that produce a rank of feasible solutions, and introduces a set of variables and constraints that provide a local maximum capacity. Thus, it is possible to observe that using the new formulation, the maximum capacity is close to the solution given by the approach 1, with 106,88 and 109,74 trains respectively.

In addition, it must be remarked that the extension B and Approach 1 provide the same traffic of trains for the shortest path $(B, 16)$ that is at the same time, the path with highest flow with 20,71 trains in one hour.

3. Single track network

In this subsection the case of a single track network is solved through the method explained in chapter 4, using only the non-linear formulation, in order to observe the overall reduction in capacity when compared to the double track case.

3.1. General results. Taking into account a single track under the same network, the capacity goes down from 1.969,85 to 602,03 trains corresponding to a 69,4% less than the network with double track (see Table 3). In other words, moving from single track to double track, increases the capacity 3,3 times, a results that is consistent with [13].

The paths with higher trains flows are (C, F) with 106,48 trains, followed by (B, C) with 99,12 trains. Conversely, the lines with lower traffic are (C, D) and $(16, B)$ with 10,2 and 12,55 trains respectively. Likewise, the gap between the results provided by both models is small. Let us note that the formulation 2, gives an optimal rank between 602,16 and 602,18 trains, and the model 1 indicates 602,03 trains in 18 hours.

The percentage of delays is only increased by 2,5% (from 40,2% to 42,5%)regarding to the double track case, being the line $(A, 24)$ the one with the highest percentage of delays with 97% that corresponds to the longest path. On the other hand, it is necessary to mention that the distribution of trains along the network in the single track case, is more homogeneous than in the double track case. For instance, if the extreme points are not considered, the traffic rank of each path is between 24-54 trains in contrast to 40-213 provided by the double track case (see Table 4).

3.2. Rolling stock. Table 8 shows the results without taking into account the parameters associated with the composition of train flows. Thus, the capacity is increased in almost 60 trains corresponding to 9,9% (from 602,03 to 661,67) and reducing the paths from 14 to 10, this is similar to the double track case, doing the same analysis. However, it must be remarked that in the double track case, where the rolling stock parameters were not included, the capacity was increased by 35,6% and in this single track case only by 9,9%.

Likewise, it is possible to see in Table 8 that the model gives priority to paths resulting in the highest train flows, as is the case for the paths $(24, F)$ and $(24, D)$ corresponding to 127,34 and 116,16 trains respectively. Nevertheless, in contrast to the double track (when the rolling stock parameters are not considered) for a single track, the paths with the highest flow change from (C, F) to $(24, F)$. On the other hand, the delays shown in this analysis decrease significantly from 42,5% (corresponding to the total trains in a network with single track and including the rolling stock parameters) to 10,79%, this is totally opposite to the result with the same analysis in the double track case (review the double track - rolling stock). Thus, for a single track network the parameters associated to the composition of train flows impact much more on the number of trains delayed than on the capacity of the railway system.

3.3. Dwell times. In Table 9 is represented the variations of the capacity and the percentage of trains delayed when the dwell times are incremented by 80% with respect to a base case or they are decrease in the same percentage. Thus, it can be seen that the capacity of the network could be increased by 39% (allowing 838,28 trains) and decrease the percentage of trains delayed to a 1% if the dwell times are reduced by 80%. In the opposite case, when the dwell times are increased by

80%, the percentage of delayed trains increases up to a 56% and the capacity of the network is reduced from 602,03 to 478,45 trains (21%). Therefore, in the single track case, a reduction of the dwell time produces a strong fall in the percentage of delays.

3.4. Headway times. Equal to the case of headway time for double track, Table 13 in appendix B, shows the results for the case when a single track network is considered. However, in this situation, the impact of the headway time is greater than in the case of double track because of two main factors. The first one is that the headway time is composed by the sections that do not have crossing loop and their respective dwell time (i.e. the third column of Table 13, associated with the dwell time, only takes into account the stations which have crossing loops, and those that do not have, are included in the headway time). And secondly, it includes, for the nodes with more than one incoming section, the maximum headway time between them (review the explanation for headway time for a single track in chapter 4). Thus, when the single track is considered, the average impact of the headway time, or a good scheduling, is 57,3% over the total traveling time.

Table 14 shows, for each path, the difference between the headway time for double and single track, where the minimum increase, for this case study, is for the line $(16, A)$ with 20%, beacuse it is has only one node, and the maximum for the path $(D, 16)$ with 115% more than the double track. Therefore, the average effect of the infrastructure parameter over the headway time is of 49%.

3.5. Using Extension B. Considering as well a period of time $T = 60$ minutes, as for the case of double track, Table 11 reports the results using the different formulation presented, including the extension B for the single track case. Particularly, the new model provides traffic only in 4 paths and a maximum capacity of 24,77 trains in one hour, that it is not close to the optimal values given for the other models. However, the four formulations agree regarding the path $(B, 16)$ with highest train flows (6,74 trains)

4. Large-scale network

The Rodalies network of Catalunya has been considered to apply the models exposed in this master thesis. The railway system has 430 km of length (R3 is taken into account until Vic station), it possesses 12 points of origin / destinations, 9 nodes and 95 stations, as it is possible to observe in the Fig 1 of the Appendix C. The time horizon used was the 18 hours, just as in previous cases, it was considered a system with double track, because currently over than 90% of the network is configured in this way. On the other hand, Table 6 (Appendix C) and Tables 15-16 (Appendix B) present the distance matrix between each station and the configuration of the numbers and name of stations respectively.

To this case of study, only the O/D pairs corresponding to operative lines in the current system have been considered, to keep a realistic situation. And, the distribution mixes of trains associated with the parameters $\eta^k_{p,q}$ y $\nu^k_{p,q}$ have not been taken into account either, to study the absolute capacity of the network.

Thus, the objective implements two different situation; the first one, a case without considering a minimum amount of trains for each operative line, and a second case, using the *Extension A* presented in the chapter 4, that includes a lower bound for train flows that travel across each line based on the current table of service on the network. The aim is to show, on one hand, how the capacity is reduced when a minimum frequency of traffic is included, and on the other hand, to evaluate the available capacity of the network considering the current traffic.

The solving methodology used in this section corresponds only with a lineal formulation, because MINOS has difficulties to arrive at an optimal solution. Moreover, to facilitate the understanding of the results, all the tables discussed below are attached in Appendix C.

4.1. General results without a lower bound. Table 1 shows the train flows for each path belonging to the system and their respective solving methodology. The results follow the same criteria of the previous case study, i.e. the model 1 provides a total number of trains between the optimal rank given by the formulation 2. Specifically, for the first model, the maximum train flows correspond to 1.338,89 which is bounded by 638,42 (Lower bound - Approach 2) and 1.623,82 trains (Upper bound - Approach 2). Furthermore, the path which has the highest traffic is (E, I) which is also the one that has the shortest path with 13,1 km.

However, a remarkable fact is that the capacity, for the case when a large network is considered, is *lower* than the previous case study which represents only a part of the Rodalies system. In other words, the first case set out, provides a maximum capacity of 1969,85 trains, and for the complete system 1.338,89 trains (i.e. a 32% less than the first case study). This situation can be supported with the results set out in Table 2, that show the percentage of delays of the network. Thus, when the small case is extended to a large scale, the kilometers of tracks that compose the system are longer and the number of nodes is increased, producing a growth of the trains delayed. Thereby, when the network is extended, the percentage of delays changes from 40,2% to 84,62%, implying a decrease in the capacity.

4.2. General results considering lower bound. By including the lower bound as a constraint (*Extension A*), the capacity goes down to 1.233,61, equivalent to a reduction of 7,86% (see Table 3). Nevertheless, not all paths are reducing their capacity in a similar percentage. Some effectively decrease their capacity, but others increase their traffic to get an optimal solution under the new scenario. The percentage of trains delayed is reduced by the effect of the new lower bound constraint from 84,62%, in the previous case, to 80,03%, being again the path with highest traffic the least congested (E, I) (see Table 4).

The second column of the Table 5, indicates the current timetable of the network (which are the parameters included in the new constraints), and the fourth column shows the results set out in Table 3, thus, in the fifth column it is possible to observe the percentage of use of each path belonging to the railway system considered. For instance, in the first line (A, C), the scheduling demands 38 trains in 18 hours, and the algorithm provides this minimum requirement, producing a 100% of the capacity. On the other hand, for the shortest path (E, I), the minimum required is

16 trains in 18 hours, and the model 1 gives a maximum capacity of 381,03 trains, that implies only 1% of use. These last extreme cases are the reason why the total using of the network is only 12,33%, being that, 11 of 18 paths are 100% used.

The third column is to compare with the minimum requirement shown in the second column. It is possible to see that only three paths do not satisfy the demand, namely (A, J), (J, A) and (D, H). However, if these three paths are not considered, the other paths would not be used to their maximum capacity.

Finally, Table 7 shows the maximum possible traffic in each section, independently of the type of train. Thus, it can be seen that the sections belonging to the path (E, I) have the highest flow of trains, followed by the sections corresponding to the core of the network. To make the interpretation of the Table 7 easier, Fig. 2, illustrates the railway system and in red the sequences with higher possible traffic. Thus, the sections which include the station of *Stants*, *Passeig de Gràcia*, *Plaza Catalunya* and *Arc de Triomf* also have a high flow of trains. Furthermore, to comment that the route from the station 15 to F, also has a lot of traffic due to that the paths $(9, F)$, (B, G), (D, G) and (L, G) cross it.

Chapter 6
Conclusions

In this master thesis, two approaches (1 and 2) have been presented based on the works done by [1] and [2] to the case of networks with a general configuration for the estimation of the maximum capacity of a railway network. These two models are based on establishing sets of restrictions that incorporate flow variables or the number of circulations in elements of a railway network during a time period of operations. The potential of these models lies in their ability to provide estimates of maximum flows taking into account only: a) their mutual interactions at specific points in the network, such as crossings or small stations, acting as causal agents that limit such flows, b) the possibilities of blocking sections or sections for security reasons. On the one hand, through model 1 it is possible to have a point estimate of the maximum capacity and also, under these conditions of maximum flows, an estimate of the fraction of the flows that are affected by blockages (delay by blockage). On the other hand, through model 2 it is possible to obtain, from two families of different restrictions, a range of values within which the maximum capacity of the system should be. The lower bound of this interval is marked by conditions established by an unfavorable scheduling, while the upper bound is determined by favorable scheduling.

Both model 1 and 2 incorporate non-linear constraints and to solve the extension made in this master's thesis of these models, a heuristic method based on fixed-point iterations has been implemented in which, in each iteration, various non-linear terms are frozen, to obtain a linear version of the problem.

The models have been described using a graph notation, to improve their understanding with respect to the proposed extensions. The computational tests have been carrying out using AMPL and CPLEX or MINOS as solvers.

In general words, approach 1 provides more information than the approach 2, allowing a better understanding of the network studied. For instance, it is possible to know the occupation percentage of each path, section, node and the amount of trains with delay that follow a path or just in a particular section. Moreover, it can be adapted easily for cases with double / single-track and give an optimal solution unlike the approach one that provides a region of solution bounded by efficient and inefficient scheduling. Nonetheless, this last parameter (timetable) is the biggest contrast between both models, because the second model does not incorporate it in an explicit way, but it leaves open the possibility to include it by means of a different estimation of the headway time.

The formulations with their respective extensions have been applied on a medium size network corresponding to the line 1,2,3 and 7 of Rodalies Network of Catalunya. Also model 1, for double track, taken into account in the extension A, has been tested on the complete railway system of Rodalies.

Given the results shown in chapter 5, applied on the case studies commented, it is possible to observe how relevant the parameters explained in the introduction are (i.e. dwell time, headway time, rolling stock and infrastructure). Each one of them has been modified, keeping the other constant, to observe the impact in percentage on the performance of the test networks. Thus, double / single-track has been considered to represent the case associated with infrastructure parameters (where in the single-track case, the station without crossing loop can play the role as signal or simple station), mix of trains to evaluate the rolling stock, dwell time, to study the time parameter and the headway time to dimension the influence of an efficient scheduling, delivering for each situation, the following summary of results:

* The models tend to prioritize the shortest paths and their traffic, providing high train flows on these paths and low delay percentages.
* A network containing long paths and with high amount of nodes, implies a growth over the percentage of delayed trains.
* For double track systems, the rolling stock parameters have an impact over 30% on the capacity of the network.
* A decrease of dwell times can affect up to 40% the amount of delayed trains and the capacity of the network. On the other hand, an increase of dwell times, can affect up to 20% the capacity and the percentage of delays.
* An efficient scheduling can decrease the travel times over 30%.
* A double track system has at least 3 times more capacity than a single track system.
* In general, the impact of the time parameters (dwell time and headway time) is higher on a network with single track than a system with double track.
* The rolling stock parameter has more impact on the percentage of trains delayed than on capacity of the network in case that a single track is considered.

Future extensions can be developed from the approaches set out in this work including the concepts of passenger demand and type of service (i.e. express, international, regional, urban). For instance, the models proposed use the distribution of train types as a fixed parameter which is given. However, that parameter can be estimated in function of the proportion of the type of service required to supply the demand for the different stations on the railway system. The equations associated with the fleet heterogeneity will continue being the same, the only change would be that now the parameters will depend on deterministic / stochastic variables related to the passenger demand. Likewise, the extension applied for the large-scale case, used only a simple constraint to force the model to have a minimum traffic in each line, nevertheless, it could also be written under demand variable, keeping the difference between both constraints, due to that one reflects how the type of service is distributed across the network and the other is associated with a minimum service offered.

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Appendix A
Formulations

Single-track non-linear problem 1

$$P^j_{i,j} \;=\; \frac{1}{T}\left[\sum_{k\in\mathbb{K}}(\alpha_{j,k}\; y^k_{i,j} \;+\widehat{y}^k_{i,j}\;\max_{(j,n)\in\mathbb{F}}\beta^k_{j,n})\right] \quad \forall j\in\mathbb{G},\ \forall (m,j)\in\mathbb{F},\ \forall (i,j)\in\mathbb{D}_{m,j}, n\neq i$$

$$\sum_{(m,j)\in\mathbb{F}}\ \sum_{(i,j)\in\mathbb{D}_{m,j}} P^j_{i,j} \;\leq 1 \qquad \forall j\in\mathbb{G}$$

$$\sum_{k\in\mathbb{K}}\ \sum_{(i,j)\in\mathbb{D}_{m,n}} ((\theta^k_{i,j}+\alpha^k_j)\ (y^k_{i,j}+\widehat{y}^k_{i,j}) + (\theta^k_{j,i}+\alpha^k_i)\ (y^k_{j,i}+\widehat{y}^k_{j,i})) \;\leq\; T \qquad \forall (m,n)\in\mathbb{F}$$

$$\widehat{y}^k_{i,j} = (y^k_{i,j}+\widehat{y}^k_{i,j})\left[\frac{P^j_{i,j}\ \pi^j_{i,j}}{(1\ -\ \pi^j_{i,j})}\right] \qquad \forall k\in\mathbb{K}, \forall j\in\mathbb{G},\ \forall (i,j)\in\mathbb{A}$$

$$y^k_{i,j}+\widehat{y}^k_{i,j} \;=\; \sum_{r\in\Omega_{(i,j)}} x^k_r \qquad \forall k\in\mathbb{K},\ \forall (i,j)\in\mathbb{A}$$

$$\widehat{x}^k_r \;=\; \sigma_r\ x^k_r \qquad \forall r\in\mathbb{C},\ \forall k\in\mathbb{K}$$

$$\widehat{y}^k_{i,j} \;\leq\; \sum_{r\in\Omega_{(i,j)}} (1\ -\ \sigma_r)\ x^k_r \qquad \forall (i,j)\in\mathbb{A},\ \forall k\in\mathbb{K}$$

$$\pi^j_{i,j} \;=\; \sum_{(i',j)\in\mathbb{A}} P^j_{(i',j)}\ -\ P^j_{i,j} \qquad \forall j\in\mathbb{G},\ \forall (i,j)\in\mathbb{A}$$

$$\sigma_r \;=\; \prod_{j\in\mathbb{N},(i,j)\in\mathbb{S}_r} \mu^j_{i,j} \qquad \forall j\in\mathbb{G},\ \forall (i,j)\in\mathbb{A}$$

$$\mu^j_{i,j} \;=\; \left[1\ -\ \frac{P^j_{i,j}\ \pi^j_{i,j}}{(1\ -\ \pi^j_{i,j})}\right] \qquad \forall j\in\mathbb{G},\ \forall (i,j)\in\mathbb{A}$$

$$\beta^k_{m,n} \;=\; \sum_{(i,j)\in\mathbb{D}_{m,n}} \theta^k_{i,j} \qquad \forall k\in\mathbb{K}, \forall (m,n)\in\mathbb{F}$$

$$x^k_r + x^k_{r'} = \eta^k_{p,q}\left[\sum_{k'\in\mathbb{K}}(x^{k'}_r\ +\ x^{k'}_{r'})\right] \qquad \forall k\in\mathbb{K},\ \forall (p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r = \nu^k_{p,q}[x^k_r\ +\ x^k_{r'}] \qquad \forall k\in\mathbb{K},\ \forall (p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r, \widehat{x}^k_r, y^k_{i,j}, \widehat{y}^k_{i,j} \geq 0$$

$$1 \geq P^j_{i,j} \geq 0$$

$$1 \geq \pi^j_{i,j} \geq 0$$

Double-track non-linear problem 1

$$P_{i,j}^{j} \;=\; \frac{1}{T}\left[\sum_{k\in\mathbb{K}}(\alpha_{j,k}\; y_{i,j}^{k} \;+\widehat{y}_{i,j}^{k}\;\max_{(j,n)\in\mathbb{A}}\theta_{j,n}^{k})\right] \;\;\forall j\in\mathbb{N},\;\forall(i,j)\in\mathbb{A}, n\neq i$$

$$\sum_{(i,j)\in\mathbb{A}} P_{i,j}^{j} \;\leq 1 \qquad \forall j\in\mathbb{N}$$

$$\sum_{k\in\mathbb{K}}(\theta_{i,j}^{k}+\alpha_{j}^{k})\;(y_{i,j}^{k}+\widehat{y}_{i,j}^{k}) \;\leq\; T \qquad \forall(i,j)\in\mathbb{A}$$

$$\widehat{y}_{i,j}^{k} = (y_{i,j}^{k}+\widehat{y}_{i,j}^{k})\left[\frac{P_{i,j}^{j}\;\pi_{i,j}^{j}}{(1\;-\;\pi_{i,j}^{j})}\right] \qquad \forall k\in\mathbb{K},\forall j\in\mathbb{N},\;\forall(i,j)\in\mathbb{A}$$

$$y_{i,j}^{k}+\widehat{y}_{i,j}^{k} \;=\; \sum_{r\in\Omega_{(i,j)}} x_{r}^{k} \qquad \forall k\in\mathbb{K},\;\forall(i,j)\in\mathbb{A}$$

$$\widehat{x}_{r}^{k} \;=\; \sigma_{r}\;x_{r}^{k} \qquad \forall r\in\mathbb{C},\;\forall k\in\mathbb{K}$$

$$\widehat{y}_{i,j}^{k} \;\leq\; \sum_{r\in\Omega_{(i,j)}}(1\;-\;\sigma_{r})\;x_{r}^{k} \qquad \forall(i,j)\in\mathbb{A},\;\forall k\in\mathbb{K}$$

$$\pi_{i,j}^{j} \;=\; \sum_{(i',j)\in\mathbb{A}} P_{(i',j)}^{j} \;-\; P_{i,j}^{j} \qquad \forall j\in\mathbb{N},\;\forall(i,j)\in\mathbb{A}$$

$$\sigma_{r} \;=\; \prod_{j\in\mathbb{N},(i,j)\in\mathbb{S}_{r}} \mu_{i,j}^{j} \qquad \forall j\in\mathbb{N},\;\forall(i,j)\in\mathbb{A}$$

$$\mu_{i,j}^{j} \;=\; \left[1\;-\;\frac{P_{i,j}^{j}\;\pi_{i,j}^{j}}{(1\;-\;\pi_{i,j}^{j})}\right] \qquad \forall j\in\mathbb{N},\;\forall(i,j)\in\mathbb{A}$$

$$x_{r}^{k}+x_{r'}^{k} = \eta_{p,q}^{k}\left[\sum_{k'\in\mathbb{K}}(x_{r}^{k'}\;+\;x_{r'}^{k'})\right] \qquad \forall k\in\mathbb{K},\;\forall(p,q)\in\mathbb{R},\;\forall r\in\xi_{p,q}$$

$$x_{r}^{k} = \nu_{p,q}^{k}[x_{r}^{k}\;+\;x_{r'}^{k}] \qquad \forall k\in\mathbb{K},\;\forall(p,q)\in\mathbb{R},\;\forall r\in\xi_{p,q}$$

$$x_{r}^{k},\widehat{x}_{r}^{k},y_{i,j}^{k},\widehat{y}_{i,j}^{k}\geq 0$$

$$1\geq P_{i,j}^{j}\geq 0$$

$$1\geq \pi_{i,j}^{j}\geq 0$$

Single-track linear problem 1

$$\sum_{k\in\mathbb{K}}\sum_{(m,n)\in\mathbb{F}}\sum_{(i,j)\in\mathbb{D}_{m,n}}(\alpha_{j,k}\ y^k_{i,j}\ +\widehat{y}^k_{i,j}\ \widehat{\beta}^k_{i,j})\le T \qquad \forall j\in\mathbb{G}$$

$$\sum_{k\in\mathbb{K}}\sum_{(i,j)\in\mathbb{D}_{m,n}}((\theta^k_{i,j}+\alpha^k_j)\ (y^k_{i,j}+\widehat{y}^k_{i,j})+(\theta^k_{j,i}+\alpha^k_i)\ (y^k_{j,i}+\widehat{y}^k_{j,i}))\ \le\ T \qquad \forall(m,n)\in\mathbb{F}$$

$$\widehat{y}^k_{i,j}=(y^k_{i,j}+\widehat{y}^k_{i,j})\left[\frac{P^j_{i,j}\ \pi^j_{i,j}}{(1\ -\ \pi^j_{i,j})}\right] \qquad \forall k\in\mathbb{K},\forall j\in\mathbb{G},\ \forall(i,j)\in\mathbb{A}$$

$$y^k_{i,j}+\widehat{y}^k_{i,j}\ =\ \sum_{r\in\Omega_{(i,j)}}x^k_r \qquad \forall k\in\mathbb{K},\ \forall(i,j)\in\mathbb{A}$$

$$\widehat{x}^k_r\ =\ \sigma_r\ x^k_r \qquad \forall r\in\mathbb{C},\ \forall k\in\mathbb{K}$$

$$\widehat{y}^k_{i,j}\ \le\ \sum_{r\in\Omega_{(i,j)}}(1\ -\ \sigma_r)\ x^k_r \qquad \forall(i,j)\in\mathbb{A},\ \forall k\in\mathbb{K}$$

$$x^k_r+x^k_{r'}=\eta^k_{p,q}\left[\sum_{k'\in\mathbb{K}}(x^{k'}_r\ +\ x^{k'}_{r'})\right] \qquad \forall k\in\mathbb{K},\ \forall(p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r=\nu^k_{p,q}[x^k_r\ +\ x^k_{r'}] \qquad \forall k\in\mathbb{K},\ \forall(p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r,\widehat{x}^k_r,y^k_{i,j},\widehat{y}^k_{i,j}\ge 0$$

Parameters

$$P^j_{i,j}\ =\ \frac{1}{T}\left[\sum_{k\in\mathbb{K}}(\alpha_{j,k}\ y^k_{i,j}\ +\widehat{y}^k_{i,j}\ \widehat{\beta}^k_{i,j})\right]\ \ \forall j\in\mathbb{G},\ \forall(m,j)\in\mathbb{F},\forall(i,j)\in\mathbb{D}_{m,j}$$

$$\beta^k_{m,n}\ =\ \sum_{(i,j)\in\mathbb{D}_{m,n}}\theta^k_{i,j} \qquad \forall k\in\mathbb{K},\forall(m,n)\in\mathbb{F}$$

$$\widehat{\beta}^k_{i,j}\ =\max_{(j,n)\in\mathbb{F}}\beta^k_{j,n} \qquad \forall j\in\mathbb{G},\ \forall(i,j)\in\mathbb{D}_{m,j},\forall(m,j)\in\mathbb{F},\forall k\in\mathbb{K},n\neq m$$

$$\pi^j_{i,j}\ =\ \sum_{(i',j)\in\mathbb{A}}P^j_{(i',j)}\ -\ P^j_{i,j} \qquad \forall j\in\mathbb{G},\ \forall(i,j)\in\mathbb{A}$$

$$\sigma_r\ =\ \prod_{j\in\mathbb{N},(i,j)\in\mathbb{S}_r}\mu^j_{i,j} \qquad \forall j\in\mathbb{G},\ \forall(i,j)\in\mathbb{A}$$

$$\mu^j_{i,j}\ =\ \left[1\ -\ \frac{P^j_{i,j}\ \pi^j_{i,j}}{(1\ -\ \pi^j_{i,j})}\right] \qquad \forall j\in\mathbb{G},\ \forall(i,j)\in\mathbb{A}$$

Double-track linear problem 1

$$\sum_{k\in\mathbb{K}}\sum_{(i,j)\in\mathbb{A}}(\alpha_{j,k}\ y^k_{i,j}\ +\widehat{y}^k_{i,j}\ \widehat{\theta}^k_{i,j})\le T \qquad \forall j\in\mathbb{N}$$

$$\sum_{k\in\mathbb{K}}(\theta^k_{i,j}+\alpha^k_j)\ (y^k_{i,j}+\widehat{y}^k_{i,j})\ \le\ T \qquad \forall (i,j)\in\mathbb{A}$$

$$\widehat{y}^k_{i,j}=(y^k_{i,j}+\widehat{y}^k_{i,j})\left[\frac{P^j_{i,j}\ \pi^j_{i,j}}{(1\ -\ \pi^j_{i,j})}\right] \qquad \forall k\in\mathbb{K},\forall j\in\mathbb{N},\ \forall (i,j)\in\mathbb{A}$$

$$y^k_{i,j}+\widehat{y}^k_{i,j}\ =\ \sum_{r\in\Omega_{(i,j)}}x^k_r \qquad \forall k\in\mathbb{K},\ \forall (i,j)\in\mathbb{A}$$

$$\widehat{x}^k_r\ =\ \sigma_r\ x^k_r \qquad \forall r\in\mathbb{C},\ \forall k\in\mathbb{K}$$

$$\widehat{y}^k_{i,j}\ \le\ \sum_{r\in\Omega_{(i,j)}}(1\ -\ \sigma_r)\ x^k_r \qquad \forall (i,j)\in\mathbb{A},\ \forall k\in\mathbb{K}$$

$$x^k_r+x^k_{r'}=\eta^k_{p,q}\left[\sum_{k'\in\mathbb{K}}(x^{k'}_r\ +\ x^{k'}_{r'})\right] \qquad \forall k\in\mathbb{K},\ \forall (p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r=\nu^k_{p,q}[x^k_r\ +\ x^k_{r'}] \qquad \forall k\in\mathbb{K},\ \forall (p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r,\widehat{x}^k_r,y^k_{i,j},\widehat{y}^k_{i,j}\ge 0$$

Parameters

$$P^j_{i,j}\ =\ \frac{1}{T}\left[\sum_{k\in\mathbb{K}}(\alpha_{j,k}\ y^k_{i,j}\ +\widehat{y}^k_{i,j}\ \widehat{\theta}^k_{i,j})\right] \qquad \forall j\in\mathbb{N},\ \forall (i,j)\in\mathbb{A}$$

$$\widehat{\theta}^k_{i,j}\ =\max_{(j,n)\in\mathbb{A}}\theta^k_{j,n} \qquad \forall j\in\mathbb{N},\ \forall (i,j)\in\mathbb{A},\forall j\in\mathbb{K},n\ne i$$

$$\pi^j_{i,j}\ =\ \sum_{(i',j)\in\mathbb{A}}P^j_{(i',j)}\ -\ P^j_{i,j} \qquad \forall j\in\mathbb{N},\ \forall (i,j)\in\mathbb{A}$$

$$\sigma_r\ =\ \prod_{j\in\mathbb{N},(i,j)\in\mathbb{S}_r}\mu^j_{i,j} \qquad \forall j\in\mathbb{N},\ \forall (i,j)\in\mathbb{A}$$

$$\mu^j_{i,j}\ =\ \left[1\ -\ \frac{P^j_{i,j}\ \pi^j_{i,j}}{(1\ -\ \pi^j_{i,j})}\right] \qquad \forall j\in\mathbb{N},\ \forall (i,j)\in\mathbb{A}$$

Single-track non-linear problem 2

$$\sum_{k\in\mathbb{K}}\sum_{(i,j)\in\mathbb{D}_{m,n}}((\theta^k_{i,j}+\alpha^k_j)\ y^k_{i,j}+(\theta^k_{j,i}+\alpha^k_i)\ y^k_{j,i})\ \leq\ T \qquad \forall(m,n)\in\mathbb{F}$$

$$\sum_{k\in\mathbb{K}}\left[\alpha^k_j y^k_{i,j}+\left\{\max_{(j,n)\in\mathbb{F}}\beta^k_{j,n}\right\}\min(Y_{i,j},Y_{j,i})\right]\leq T \qquad \forall j\in\mathbb{G},\forall(m,j)\in\mathbb{F},\forall(i,j)\in\mathbb{D}_{m,j}$$

$$\sum_{k\in\mathbb{K}}(\alpha^k_j y^k_{i,j})\leq T \qquad \forall k\in\mathbb{K},\forall(m,j)\in\mathbb{F},\forall(i,j)\in\mathbb{D}_{m,j}$$

$$y^k_{i,j}=\sum_{r\in\Omega_{(i,j)}}x^k_r \qquad \forall k\in\mathbb{K},\forall(i,j)\in\mathbb{A}$$

$$Y_{i,j}=\sum_{k\in\mathbb{A}}y^k_{i,j} \qquad \forall(i,j)\in\mathbb{A}$$

$$x^k_r+x^k_{r'}=\eta^k_{p,q}\left[\sum_{k'\in\mathbb{K}}(x^{k'}_r\ +\ x^{k'}_{r'})\right] \qquad \forall k\in\mathbb{K},\ \forall(p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r=\nu^k_{p,q}[x^k_r\ +\ x^k_{r'}] \qquad \forall k\in\mathbb{K},\ \forall(p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r,y^k_{i,j}\geq 0$$

Double-track linear problem 2

$$\sum_{k \in \mathbb{K}} (\theta_{i,j}^k + \alpha_j^k)\ y_{i,j}^k \leq\ T \qquad \forall (i,j) \in \mathbb{A}$$
$$\sum_{k \in \mathbb{K}} \sum_{(i,j) \in \mathbb{A}} (\alpha_{j,k}\ y_{i,j}^k\ + y_{i,j}^k\ \widehat{\theta}_{i,j}^k) \leq T \qquad \forall j \in \mathbb{N}$$
$$\sum_{k \in \mathbb{K}} (\alpha_j^k y_{i,j}^k) \leq T \qquad \forall (i,j) \in \mathbb{A}$$
$$y_{i,j}^k = \sum_{r \in \Omega_{(i,j)}} x_r^k \qquad \forall k \in \mathbb{K}, \forall (i,j) \in \mathbb{A}$$
$$Y_{i,j} = \sum_{k \in \mathbb{A}} y_{i,j}^k \qquad \forall (i,j) \in \mathbb{A}$$
$$x_r^k + x_{r'}^k = \eta_{p,q}^k \left[\sum_{k' \in \mathbb{K}} (x_r^{k'}\ +\ x_{r'}^{k'}) \right] \qquad \forall k \in \mathbb{K},\ \forall (p,q) \in \mathbb{R},\ \forall r \in \xi_{p,q}$$
$$x_r^k = \nu_{p,q}^k [x_r^k\ +\ x_{r'}^k] \qquad \forall k \in \mathbb{K},\ \forall (p,q) \in \mathbb{R},\ \forall r \in \xi_{p,q}$$
$$x_r^k, y_{i,j}^k \geq 0$$

Parameters

$$\widehat{\theta}_{i,j}^k\ = \max_{(j,n) \in \mathbb{A}} \theta_{j,n}^k \qquad \forall j \in \mathbb{N},\ \forall (i,j) \in \mathbb{A}, \forall j \in \mathbb{K}, n \neq i$$

Single-track linear problem 2

$$\sum_{k\in\mathbb{K}} \sum_{(i,j)\in\mathbb{D}_{m,n}} ((\theta^k_{i,j}+\alpha^k_j)\ y^k_{i,j} + (\theta^k_{j,i}+\alpha^k_i)\ y^k_{j,i})\ \le\ T \qquad \forall (m,n)\in\mathbb{F}$$

$$\sum_{k\in\mathbb{K}} \left[\alpha^k_j\ y^k_{i,j}\ +\ \widehat{\beta}^k_{i,j}\ Q_{i,j}\right]\ \le\ T \qquad \forall k\in\mathbb{K}, \forall (m,j)\in\mathbb{F}, \forall (i,j)\in\mathbb{D}_{m,j}$$

$$\sum_{k\in\mathbb{K}} (\alpha^k_j y^k_{i,j}) \le T \qquad \forall k\in\mathbb{K}, \forall (m,j)\in\mathbb{F}, \forall (i,j)\in\mathbb{D}_{m,j}$$

$$Q_{i,j}\ \le\ Y_{i,j} \qquad \forall k\in\mathbb{K}, \forall (i,j)\in\mathbb{A}$$

$$Q_{i,j}\ \le\ Y_{j,i} \qquad \forall k\in\mathbb{K}, \forall (i,j)\in\mathbb{A}$$

$$d1_{i,j}\ +\ d2_{i,j}\ =\ 1 \qquad \forall k\in\mathbb{K}, \forall (i,j)\in\mathbb{A}$$

$$Q_{i,j}\ \ge\ Y_{i,j}\ -\ M(\ 1-d1_{i,j}) \qquad \forall k\in\mathbb{K}, \forall (i,j)\in\mathbb{A}$$

$$Q_{i,j}\ \ge\ Y_{j,i}\ -\ M(\ 1-d2_{i,j}) \qquad \forall k\in\mathbb{K}, \forall (i,j)\in\mathbb{A}$$

$$y^k_{i,j} = \sum_{r\in\Omega_{(i,j)}} x^k_r \qquad \forall k\in\mathbb{K}, \forall (i,j)\in\mathbb{A}$$

$$Y_{i,j} = \sum_{k\in\mathbb{A}} y^k_{i,j} \qquad \forall (i,j)\in\mathbb{A}$$

$$x^k_r + x^k_{r'} = \eta^k_{p,q}\left[\sum_{k'\in\mathbb{K}} (x^{k'}_r\ +\ x^{k'}_{r'})\right] \qquad \forall k\in\mathbb{K},\ \forall (p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r = \nu^k_{p,q}[x^k_r\ +\ x^k_{r'}] \qquad \forall k\in\mathbb{K},\ \forall (p,q)\in\mathbb{R},\ \forall r\in\xi_{p,q}$$

$$x^k_r, y^k_{i,j} \ge 0$$

$$d1_{i,j}\in\{0,1\}$$

$$d2_{i,j}\in\{0,1\}$$

Parameters

$$\beta^k_{m,n}\ =\ \sum_{(i,j)\in\mathbb{D}_{m,n}} \theta^k_{i,j} \qquad \forall k\in\mathbb{K}, \forall (m,n)\in\mathbb{F}$$

$$\widehat{\beta}^k_{i,j}\ =\ \max_{(j,n)\in\mathbb{F}} \beta^k_{j,n} \qquad \forall j\in\mathbb{G},\ \forall (i,j)\in\mathbb{D}_{m,j}, \forall (m,j)\in\mathbb{F}, \forall j\in\mathbb{K}, n\ne m$$

Appendix B

Tables for double and single track network

	Linear Problem			Non-Linear
Corridors	App 2	LB App 1	UB App 1	App 2
$(16, A)$	$159, 68$	$80, 93$	$133, 79$	$153, 6$
$(16, B)$	$193, 15$	$144, 45$	$193, 15$	$193, 15$
$(16, D)$	0	0	$15, 85$	0
$(16, E)$	$111, 31$	$82, 84$	$221, 51$	$210, 33$
$(24, D)$	$212, 99$	$174, 14$	$123, 29$	$211, 75$
$(24, E)$	$100, 22$	0	0	$10, 17$
$(24, F)$	0	0	0	$1, 08$
$(A, 16)$	$98, 33$	$49, 84$	$82, 39$	$94, 58$
(A, C)	$64, 49$	$39, 4$	$127, 64$	$79, 33$
$(B, 16)$	$372, 78$	$278, 78$	$372, 78$	$372, 78$
(C, A)	$27, 12$	$16, 56$	$53, 67$	$33, 36$
(C, E)	$40, 16$	$24, 97$	$34, 08$	$34, 69$
(C, F)	0	0	$239, 13$	0
$(D, 16)$	0	0	$16, 17$	0
$(D, 24)$	$338, 79$	$276, 99$	$196, 11$	$336, 82$
$(E, 16)$	$56, 12$	$41, 77$	$111, 69$	$106, 05$
$(E, 24)$	$31, 2$	0	0	$3, 16$
(E, C)	$163, 51$	$101, 67$	$138, 73$	$141, 24$
$(F, 24)$	0	$1, 74$	0	$1, 9$
(F, C)	0	0	$121, 55$	0
$TOTAL$	$1969, 85$	$1314, 08$	$2181, 53$	$1983, 99$

TABLE 1. Double track network results

	Linear Problem				
Corridors	Path	Total Trains	Trains without Delay	% Delayed	% without Delay
(A, C)	2	64	3	95%	5%
$(A, 16)$	6	98	74	24%	76%
$(B, 16)$	12	373	303	19%	81%
(C, A)	14	27	1	96%	4%
$(16, A)$	19	160	137	14%	86%
$(16, B)$	25	193	164	15%	85%
(C, E)	27	40	5	88%	13%
$(E, 16)$	32	56	32	43%	57%
$(E, 24)$	33	31	0	100%	0%
$(24, D)$	36	213	147	31%	69%
(E, C)	38	164	47	71%	29%
$(16, E)$	43	111	31	72%	28%
$(24, E)$	44	100	0	100%	0%
$(D, 24)$	47	339	234	31%	69%
$Total$		1969	1178	40,2%	59,8%

TABLE 2. Delayed trains considering double track

	Non-Linear Problem		
Corridors	App 2	UB App 1	LB App 1
$(16, B)$	12,55	62,9	62,9
$(16, E)$	52,82	52,82	52,82
$(24, A)$	38,34	38,34	38,34
$(24, D)$	28,36	19,75	23,11
$(A, 24)$	33,68	33,68	33,68
$(B, 16)$	24,23	121,4	121,4
(B, C)	99,12	0	0
(C, B)	48,31	0	0
(C, D)	10,2	17,28	14,52
(C, F)	106,48	106,48	106,48
$(D, 24)$	45,12	31,41	36,76
(D, C)	22,06	37,36	31,39
$(E, 16)$	26,63	26,63	26,63
(F, C)	54,13	54,13	54,13
$Total$	602,03	602,18	602,16

TABLE 3. Results considering single track case

	Non-Linear Problem				
Corridors	Path	Total Trains	Trains without Delay	% Delayed	% without Delay
$(A, 24)$	7	$33, 68$	$0, 9$	97%	3%
(B, C)	8	$99, 12$	$38, 51$	61%	39%
$(B, 16)$	12	$24, 23$	$23, 71$	2%	98%
$(24, A)$	20	$38, 34$	$13, 03$	66%	34%
(C, B)	21	$48, 31$	$15, 78$	67%	33%
$(16, B)$	25	$12, 55$	$12, 53$	0%	100%
(C, F)	28	$106, 48$	$77, 62$	27%	73%
(C, D)	29	$10, 2$	$7, 47$	27%	73%
$(E, 16)$	32	$26, 63$	$10, 31$	61%	39%
$(24, D)$	36	$28, 36$	$27, 22$	4%	96%
(F, C)	39	$54, 13$	$40, 27$	26%	74%
(D, C)	40	$22, 06$	$16, 46$	25%	75%
$(16, E)$	43	$52, 82$	$20, 94$	6%	4%
$(D, 24)$	47	$45, 12$	$41, 36$	8%	92%
$Total$		$602, 03$	$346, 11$	$42, 5$%	$57, 5$%

TABLE 4. Delayed trains for single track case

	Linear Problem				
Corridors	Path	Total Trains	without Delay	% Delayed	% without Delay
$(A, 16)$	6	149	87	42%	58%
$(A, 24)$	7	76	7	91%	9%
$(B, 16)$	12	336	74	78%	22%
$(16, A)$	19	208	145	30%	70%
$(16, B)$	25	554	228	59%	41%
$(24, D)$	36	480	266	45%	55%
(E, C)	38	263	85	68%	32%
$(24, E)$	44	263	85	68%	32%
$(D, 24)$	47	118	56	53%	47%
$(F, 24)$	48	225	115	49%	51%
$Total$	282	2672	1148	58%	42%

TABLE 5. Delayed trains without considering train mixes

	Linear Problem				
Corridors	Path	Total Trains	without Delay	% Delayed	% without Delay
(A, B)	1	2	1	50%	50%
(A, C)	2	5	1	80%	20%
(A, E)	3	5	1	80%	20%
(A, F)	4	4	1	75%	25%
(A, D)	5	12	2	83%	17%
$(A, 16)$	6	102	53	48%	52%
$(A, 24)$	7	2	1	50%	50%
(B, C)	8	5	1	80%	20%
(B, E)	9	5	1	80%	20%
(B, F)	10	6	1	83%	17%
(B, D)	11	3	0	100%	0%
$(B, 16)$	12	346	234	32%	68%
$(B, 24)$	13	4	1	75%	25%
(C, A)	14	2	0	100%	0%
(B, A)	15	3	1	67%	33%
(E, A)	16	5	1	89%	20%
(F, A)	17	3	0	100%	0%
(D, A)	18	6	1	83%	17%
$(16, A)$	19	166	128	23%	77%
$(24, A)$	20	2	0	100%	0%
(C, B)	21	3	0	100%	0%
(E, B)	22	3	0	100%	0%
(F, B)	23	6	0	100%	0%
(D, B)	24	2	0	100%	0%
$(16, B)$	25	179	151	16%	84%
$(24, B)$	26	2	0	100%	0%
(C, E)	27	16	1	94%	6%
(C, F)	28	4	0	100%	0%
(C, D)	29	3	0	100%	0%
(E, D)	30	14	2	86%	14%
(E, F)	31	4	1	75%	25%
$(E, 16)$	32	90	15	83%	17%
$(E, 24)$	33	11	3	73%	27%
$(16, F)$	34	4	1	75%	25%
$(16, D)$	35	4	1	75%	25%

TABLE 6. Delayed trains considering a minimum train flows

	Linear Problem				
Corridors	Path	Total Trains	without Delay	% Delayed	% without Delay
$(24, D)$	36	178	122	31%	69%
$(24, F)$	37	3	2	33%	67%
(E, C)	38	66	20	70%	30%
(F, C)	39	2	0	100%	0%
(D, C)	40	6	1	83%	17%
(D, E)	41	12	1	92%	8%
(F, E)	42	3	0	100%	0%
$(16, E)$	43	178	51	71%	29%
$(24, E)$	44	36	6	83%	17%
$(F, 16)$	45	5	0	100%	0%
$(D, 16)$	46	4	0	100%	0%
$(D, 24)$	47	284	188	34%	66%
$(F, 24)$	48	5	4	20%	80%
Total		1808	994	45%	55%

TABLE 7. Delayed trains

	Non-linear Problem				
Corridors	Path	Total Trains	without Delay	% Delayed	% without Delay
(C, A)	14	44, 32	36, 47	18%	82%
$(16, A)$	19	35, 37	31, 27	12%	88%
(C, B)	21	205, 7	169, 27	18%	82%
(C, E)	27	2, 93	2, 82	4%	96%
(C, D)	29	1, 51	1, 45	4%	96%
$(16, F)$	34	46, 84	37, 19	21%	79%
$(24, D)$	36	116, 16	116, 16	0%	100%
$(24, F)$	37	127, 34	127, 34	0%	100%
$(16, E)$	43	63, 18	50, 17	21%	79%
$(24, E)$	44	18, 25	18, 06	1%	99%
Total		661, 67	590, 27	10, 79%	89, 21%

TABLE 8. Delayed trains for single track case without considering mixture of trains parameters

	Non-linear Problem			
Dwell times	Capacity	% Train Delayed	% capacity	% delayed
−80%	838, 26	1%	39%	99%
−50%	727, 27	28%	21%	34%
−25%	657, 9	34%	9%	19%
0	602, 03	43%	0%	0%
25%	553, 03	46%	−8%	−9%
50%	517, 53	47%	−14%	−10%
80%	478, 45	56%	−21%	−32%

TABLE 9. Effect of the variation of dwell times for single track case

	Linear Problem			
Path	Extension B	Approach 1	LB Approach 2	UB Approach 2
$(16, A)$	$5, 45$	$8, 7$	$4, 45$	$7, 43$
$(16, B)$	$10, 73$	$10, 73$	$8, 02$	$10, 73$
$(16, D)$	0	0	0	$0, 88$
$(16, E)$	$10, 02$	$6, 18$	$4, 65$	$12, 31$
$(24, A)$	$2, 5$	0	0	0
$(24, D)$	$7, 15$	$11, 83$	$9, 67$	$6, 85$
$(24, E)$	0	$5, 57$	0	0
$(24, F)$	0	0	$0, 1$	$0, 1$
$(A, 16)$	$3, 36$	$5, 36$	$2, 74$	$4, 58$
$(A, 24)$	$2, 19$	0	0	0
(A, C)	$5, 9$	4	$2, 24$	$7, 09$
$(B, 16)$	$20, 71$	$20, 71$	$15, 49$	$20, 71$
(C, A)	$2, 48$	$1, 68$	$0, 94$	$2, 98$
(C, E)	0	$2, 23$	$1, 38$	$1, 89$
(C, F)	$8, 96$	0	0	$13, 28$
$(D, 16)$	0	0	0	$0, 9$
$(D, 24)$	$11, 37$	$18, 82$	$15, 39$	$10, 89$
$(E, 16)$	$5, 05$	$3, 12$	$2, 35$	$6, 2$
$(E, 24)$	0	$1, 73$	0	0
(E, C)	0	$9, 08$	$5, 62$	$7, 71$
(E, F)	$3, 81$	0	0	0
$(F, 24)$	0	0	$0, 17$	$0, 17$
(F, C)	$4, 56$	0	0	$6, 75$
(F, E)	$2, 64$	0	0	0
Total	$106, 88$	$109, 74$	$73, 21$	$121, 45$

TABLE 10. Double track results using Extension B for T = 60

	Linear Problem			
Path	Extension B	Approach 1	LB Approach 2	UB Approach 2
$(16, B)$	$3, 49$	$3, 49$	$3, 49$	$3, 49$
$(16, E)$	0	$2, 93$	$2, 93$	$2, 93$
$(24, A)$	0	$2, 13$	$2, 13$	$2, 13$
$(24, D)$	0	$1, 1$	$1, 28$	$1, 1$
$(A, 24)$	0	$1, 87$	$1, 87$	$1, 87$
$(B, 16)$	$6, 74$	$6, 74$	$6, 74$	$6, 74$
(C, D)	$4, 6$	$0, 96$	$0, 81$	$0, 96$
(C, F)	0	$5, 92$	$5, 92$	$5, 92$
$(D, 24)$	0	$1, 74$	$2, 04$	$1, 74$
(D, C)	$9, 94$	$2, 08$	$1, 74$	$2, 08$
$(E, 16)$	0	$1, 48$	$1, 48$	$1, 48$
(F, C)	0	$3, 01$	$3, 01$	$3, 01$
Total	$24, 77$	$33, 45$	$33, 44$	$33, 45$

TABLE 11. Single track results using Extension B for T = 60

	Linear Problem					
Path	Hdway time	Dwell time	Travel time	Min time	Max time	(%)Hdway Time effect
(A, B)	30, 85	10, 7	25, 3	36	66, 85	46%
(A, C)	29, 35	10	24	34	63, 35	46%
(A, E)	46, 95	13, 4	37, 45	50, 85	97, 8	48%
(A, F)	46, 4	16, 9	36, 85	53, 75	100, 15	46%
(A, D)	39, 05	16, 3	31, 75	48, 05	87, 1	45%
$(A, 16)$	24, 7	8, 5	20, 3	28, 8	53, 5	46%
$(A, 24)$	30, 55	11, 5	25, 2	36, 7	67, 25	45%
(B, C)	12, 7	7, 7	10, 4	18, 1	30, 8	41%
(B, E)	30, 3	11, 1	23, 85	34, 95	65, 25	46%
(B, F)	29, 75	14, 6	23, 25	37, 85	67, 6	44%
(B, D)	22, 4	14	18, 15	32, 15	54, 55	41%
$(B, 16)$	5, 6	3, 7	5	8, 7	14, 3	39%
$(B, 24)$	13, 9	9, 2	11, 6	20, 8	34, 7	40%
(C, A)	31, 8	10	24	34	65, 8	48%
(B, A)	32, 5	10, 7	25, 3	36	68, 5	47%
(E, A)	47, 55	13, 4	37, 45	50, 85	98, 4	48%
(F, A)	45, 75	16, 9	36, 85	53, 75	99, 5	46%
(D, A)	40, 85	16, 3	31, 75	48, 05	88, 9	46%
$(16, A)$	26, 9	7	20, 3	27, 3	54, 2	50%
$(24, A)$	31, 8	10	25, 2	35, 2	67	47%
(C, B)	13, 5	7, 7	10, 4	18, 1	31, 6	43%
(E, B)	29, 25	11, 1	23, 85	34, 95	64, 2	46%
(F, B)	27, 45	14, 6	23, 25	37, 85	65, 3	42%
(D, B)	22, 55	14	18, 15	32, 15	54, 7	41%
$(16, B)$	6, 15	2, 2	5	7, 2	13, 35	46%
$(24, B)$	13, 5	7, 7	11, 6	19, 3	32, 8	41%
(C, E)	21	4, 4	15, 65	20, 05	41, 05	51%
(C, F)	20, 45	7, 9	15, 05	22, 95	43, 4	47%
(C, D)	13, 1	7, 3	9, 95	17, 25	30, 35	43%
(E, D)	28, 85	10, 7	23, 4	34, 1	62, 95	46%
(E, F)	36, 2	11, 3	28, 5	39, 8	76	48%
$(E, 16)$	23, 1	8, 9	18, 85	27, 75	50, 85	45%
$(E, 24)$	20, 35	5, 9	16, 85	22, 75	43, 1	47%
$(16, F)$	24, 15	10, 9	18, 25	29, 15	53, 3	45%
$(16, D)$	16, 8	10, 3	13, 15	23, 45	40, 25	42%
$(24, D)$	8, 5	4, 8	6, 55	11, 35	19, 85	43%
$(24, F)$	15, 85	5, 4	11, 65	17, 05	32, 9	48%
(E, C)	19, 15	4, 4	15, 65	20, 05	39, 2	49%
(F, C)	17, 35	7, 9	15, 05	22, 95	40, 3	43%
(D, C)	12, 45	7, 3	9, 95	17, 25	29, 7	42%
(D, E)	30, 05	10, 7	23, 4	34, 1	64, 15	47%
(F, E)	34, 95	11, 3	28, 5	39, 8	74, 75	47%
$(16, E)$	24, 7	7, 4	18, 85	26, 25	50, 95	48%
$(24, E)$	21	4, 4	16, 85	21, 25	42, 25	50%
$(F, 16)$	21, 3	12, 4	18, 25	30, 65	51, 95	41%
$(D, 16)$	16, 4	11, 8	13, 15	24, 95	41, 35	40%
$(D, 24)$	9, 05	6, 3	6, 55	12, 85	21, 9	41%
$(F, 24)$	13, 95	6, 9	11, 65	18, 55	32, 5	43%

TABLE 12. Headway time effects for double track test network

	Linear Problem					
Path	Hdway time	Dwell time	Travel time	Min time	Max time	Hdway Time effect
(A,B)	$41,45$	$5,1$	$25,3$	$30,4$	$71,85$	58%
(A,C)	42	6	24	30	72	58%
(A,E)	$58,95$	7	$37,45$	$44,45$	$103,4$	57%
(A,F)	$63,8$	$11,1$	$36,85$	$47,95$	$111,75$	57%
(A,D)	$57,5$	$11,1$	$31,75$	$42,85$	$100,35$	57%
$(A,16)$	$34,85$	$4,5$	$20,3$	$24,8$	$59,65$	58%
$(A,24)$	$46,4$	$7,5$	$25,2$	$32,7$	$79,1$	59%
(B,C)	$25,25$	$6,1$	$10,4$	$16,5$	$41,75$	60%
(B,E)	$42,2$	$7,1$	$23,85$	$30,95$	$73,15$	58%
(B,F)	$47,05$	$11,2$	$23,25$	$34,45$	$81,5$	58%
(B,D)	$40,75$	$11,2$	$18,15$	$29,35$	$70,1$	58%
$(B,16)$	$8,8$	$2,1$	5	$7,1$	$15,9$	55%
$(B,24)$	$29,65$	$7,6$	$11,6$	$19,2$	$48,85$	61%
(C,A)	$42,05$	6	24	30	$72,05$	58%
(B,A)	$41,2$	$5,1$	$25,3$	$30,4$	$71,6$	58%
(E,A)	$60,2$	7	$37,45$	$44,45$	$104,65$	58%
(F,A)	$65,05$	$11,1$	$36,85$	$47,95$	113	58%
(D,A)	$58,3$	$11,1$	$31,75$	$42,85$	$101,15$	58%
$(16,A)$	$32,4$	3	$20,3$	$23,3$	$55,7$	58%
$(24,A)$	$43,25$	6	$25,2$	$31,2$	$74,45$	58%
(C,B)	$25,55$	$6,1$	$10,4$	$16,5$	$42,05$	61%
(E,B)	$43,7$	$7,1$	$23,85$	$30,95$	$74,65$	59%
(F,B)	$48,55$	$11,2$	$23,25$	$34,45$	83	58%
(D,B)	$41,8$	$11,2$	$18,15$	$29,35$	$71,15$	59%
$(16,B)$	$6,6$	$0,6$	5	$5,6$	$12,2$	54%
$(24,B)$	$26,75$	$6,1$	$11,6$	$17,7$	$44,45$	60%
(C,E)	$21,6$	2	$15,65$	$17,65$	$39,25$	55%
(C,F)	$26,45$	$6,1$	$15,05$	$21,15$	$47,6$	56%
(C,D)	$20,15$	$6,1$	$9,95$	$16,05$	$36,2$	56%
(E,D)	$38,3$	$7,1$	$23,4$	$30,5$	$68,8$	56%
(E,F)	$44,6$	$7,1$	$28,5$	$35,6$	$80,2$	56%
$(E,16)$	$37,1$	$6,5$	$18,85$	$25,35$	$62,45$	59%
$(E,24)$	$27,2$	$3,5$	$16,85$	$20,35$	$47,55$	57%
$(16,F)$	$38,25$	$9,1$	$18,25$	$27,35$	$65,6$	58%
$(16,D)$	$31,95$	$9,1$	$13,15$	$22,25$	$54,2$	59%
$(24,D)$	$11,1$	$3,6$	$6,55$	$10,15$	$21,25$	52%
$(24,F)$	$17,4$	$3,6$	$11,65$	$15,25$	$32,65$	53%
(E,C)	$22,8$	2	$15,65$	$17,65$	$40,45$	56%
(F,C)	$27,65$	$6,1$	$15,05$	$21,15$	$48,8$	57%
(D,C)	$20,9$	$6,1$	$9,95$	$16,05$	$36,95$	57%
(D,E)	$37,85$	$7,1$	$23,4$	$30,5$	$68,35$	55%
(F,E)	$44,6$	$7,1$	$28,5$	$35,6$	$80,2$	56%
$(16,E)$	$33,4$	5	$18,85$	$23,85$	$57,25$	58%
$(24,E)$	$22,8$	2	$16,85$	$18,85$	$41,65$	55%
$(F,16)$	$41,95$	$10,6$	$18,25$	$28,85$	$70,8$	59%
$(D,16)$	$35,2$	$10,6$	$13,15$	$23,75$	$58,95$	60%
$(D,24)$	$15,05$	$5,1$	$6,55$	$11,65$	$26,7$	56%
$(F,24)$	$21,8$	$5,1$	$11,65$	$16,75$	$38,55$	57%

TABLE 13. Headway time effects for single track test network

	Linear Problem		
Path	Double track	Single track	% Increase
(A, B)	$30, 85$	$41, 45$	34%
(A, C)	$29, 35$	42	43%
(A, E)	$46, 95$	$58, 95$	26%
(A, F)	$46, 4$	$63, 8$	38%
(A, D)	$39, 05$	$57, 5$	47%
$(A, 16)$	$24, 7$	$34, 85$	41%
$(A, 24)$	$30, 55$	$46, 4$	52%
(B, C)	$12, 7$	$25, 25$	99%
(B, E)	$30, 3$	$42, 2$	39%
(B, F)	$29, 75$	$47, 05$	58%
(B, D)	$22, 4$	$40, 75$	82%
$(B, 16)$	$5, 6$	$8, 8$	57%
$(B, 24)$	$13, 9$	$29, 65$	113%
(C, A)	$31, 8$	$42, 05$	32%
(B, A)	$32, 5$	$41, 2$	27%
(E, A)	$47, 55$	$60, 2$	27%
(F, A)	$45, 75$	$65, 05$	42%
(D, A)	$40, 85$	$58, 3$	43%
$(16, A)$	$26, 9$	$32, 4$	20%
$(24, A)$	$31, 8$	$43, 25$	36%
(C, B)	$13, 5$	$25, 55$	89%
(E, B)	$29, 25$	$43, 7$	49%
(F, B)	$27, 45$	$48, 55$	77%
(D, B)	$22, 55$	$41, 8$	85%
$(16, B)$	$6, 15$	$6, 6$	7%
$(24, B)$	$13, 5$	$26, 75$	98%
(C, E)	21	$21, 6$	3%
(C, F)	$20, 45$	$26, 45$	29%
(C, D)	$13, 1$	$20, 15$	54%
(E, D)	$28, 85$	$38, 3$	33%
(E, F)	$36, 2$	$44, 6$	23%
$(E, 16)$	$23, 1$	$37, 1$	61%
$(E, 24)$	$20, 35$	$27, 2$	34%
$(16, F)$	$24, 15$	$38, 25$	58%
$(16, D)$	$16, 8$	$31, 95$	90%
$(24, D)$	$8, 5$	$11, 1$	31%
$(24, F)$	$15, 85$	$17, 4$	10%
(E, C)	$19, 15$	$22, 8$	19%
(F, C)	$17, 35$	$27, 65$	59%
(D, C)	$12, 45$	$20, 9$	68%
(D, E)	$30, 05$	$37, 85$	26%
(F, E)	$34, 95$	$44, 6$	28%
$(16, E)$	$24, 7$	$33, 4$	35%
$(24, E)$	21	$22, 8$	9%
$(F, 16)$	$21, 3$	$41, 95$	97%
$(D, 16)$	$16, 4$	$35, 2$	115%
$(D, 24)$	$9, 05$	$15, 05$	66%
$(F, 24)$	$13, 95$	$21, 8$	56%

TABLE 14. Comparative Headway time between Table 13 and Table 14

Network configuration					
Name	Large	Medium	Name	Large	Medium
Calafell	1	–	*Breda*	40	–
Segur de Calafell	2	–	*Gualba*	41	–
Cunit	3	–	*Sant Celoni*	42	–
Cubelles	4	*A*	*Palautordera*	43	–
Vilanova	5	1	Llinar del Vallès	44	–
Sitges	6	2	*Cardedeu*	45	–
Garraf	7	3	*Les Franqueses Granollers*	46	–
Platja de Castelldefels	8	4	Montmeló	47	–
Castelldefels	9	5	*Mollet Sant Fost*	48	–
Gavà	10	6	*La Llagosta*	49	–
Viladecans	11	7	*Montcada i Reixac*	50	–
Bellvitge	12	8	*Sant Andreu Comtal*	51	–
Sants	13	10	Cornellà	52	15
Passeig de Gràcia	14	11	*Sant Joan*	53	14
El clot	15	–	*Sant Feliu*	54	13
Pl. Catalunya	16	–	*El Papiol*	55	–
Arc de Triomf	17	–	*Castellbisbal*	56	–
Sant Adrià	18	19	*Gelida*	58	–
Badalona	19	20	Sant Sadurní	59	–
Montgat	20	–	*Lavern*	60	–
Montgat Nord	21	–	*La Granada*	61	–
El Masnou	22	21	*Vilafranca*	62	–
Ocata	23	–	*Els Monjos*	63	–
Premià de Mar	24	22	L'Arboç	64	–
Vilassar de Mar	25	–	*El Vendrell*	65	–
Cabrera de Mar	26	23	Rubí	66	–
Mataró	27	*E*	*Sant Cugat*	67	*D*
Sant Andreu de Llavaneres	28	–	Cerdanyola del Vallès	68	34
Caldes	29	–	Santa María	69	33
Arenys de Mar	30	–	*Manresa*	70	32
Canet de Mar	31	–	Montcada Bifurcació	71	26
Sant Pol del Mar	32	–	Torre del Baró	72	25
Calella	33	–	*Montcada Ripollet*	73	28
Pineda de Mar	34	–	Santa Perpètua	74	29
Santa Susanna	35	–	*Mollet Santa Rosa*	75	30
Malgrat de Mar	36	–	Parets del Vallès	76	31
Blanes	37	–	*Granollers*	77	*F*
Tordera	38	–	*Les Franqueses*	78	–
Hostatric	39	–	*La Garriga*	79	–

TABLE 15. Node numbers in the test network

Network configuration		
Name	Large	Medium
Figaró	80	–
Sant Martí de Centelles	81	–
Centelles	82	–
Balenyà	83	–
Tona Seva	84	–
Sabadell sud	85	–
Sabadell centre	86	–
Sabadell nord	87	–
Terrassa Est.	88	–
Terrassa	89	–
Sant Miquel	90	–
Viladecavalls	91	–
$Vacarisses - Torr$	92	–
Vacarisses	93	–
Montserrat	94	–
Sant Vicenç	95	–
St. Vicenç de Calders	A	–
Aeroport	B	–
Estació de França	C	–
$L'Hospitalet$	D	16
Sant Andreu	E	24
Granollers Centre	F	–
Maçanet-Massanes	G	–
VIC	H	–
Cerdanyola Universitat	I	–
Manresa	J	–
Martorell	K	–
Molins de Rei	L	B
El Prat	$N1$	9
Node	$N2$	–
Node	$N3$	12
Node	$N4$	–
Node	$N5$	–
Node	$N6$	–
Node	$N7$	27
Node	$N8$	–
Node	$N9$	–

TABLE 16. Node numbers in the test network

Appendix C
Catalunya Rodalies Network

Rodalies Network of Catalunya has been depicted schematically in Fig 1, where, the origin / destination pairs are represented by a big dot, medium size node means a station with crossing loop, and a little dot is for the station without crossing loop, that in a theoretical exercise could be taken as signals.

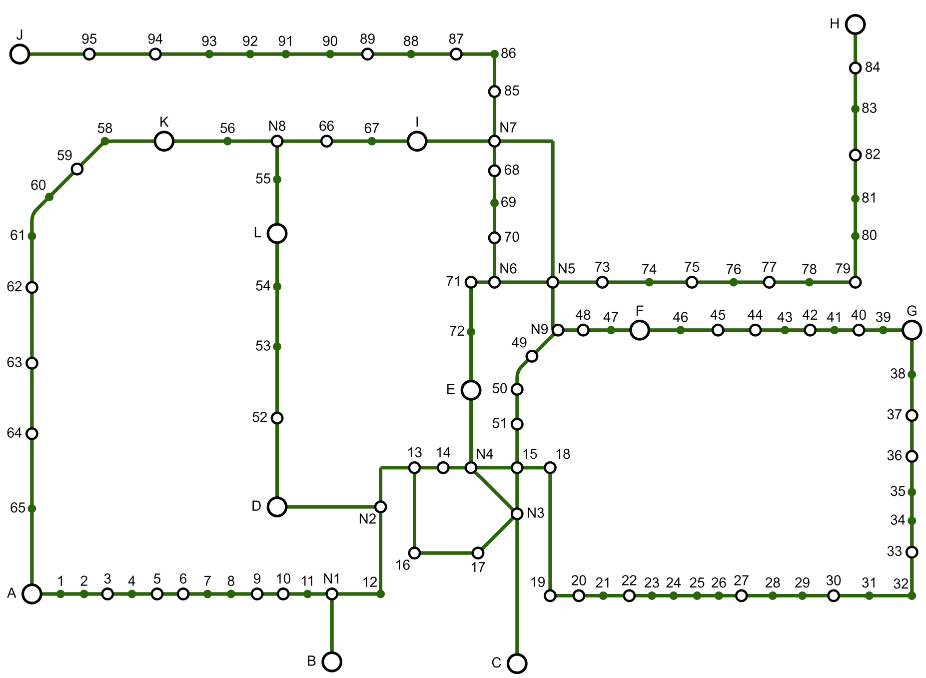

Fig. 1. Catalunya Rodalies Network

	Linear problem		
Corridors	App 1	LB App 2	UB App 2
$(9, F)$	$55, 14$	$11, 61$	$186, 24$
(A, C)	$102, 71$	$91, 84$	$31, 94$
(A, J)	0	0	0
(B, G)	$72, 56$	0	$6, 87$
(C, A)	$86, 34$	$91, 84$	$121, 54$
(D, G)	$59, 88$	$49, 09$	$162, 39$
(D, H)	$0, 6$	0	$1, 58$
(E, I)	$381, 03$	$72, 4$	$347, 24$
$(F, 9)$	$10, 88$	$11, 61$	$14, 74$
(F, K)	$63, 85$	$94, 27$	$186, 24$
(G, B)	$34, 42$	0	$6, 87$
(G, D)	$20, 3$	$49, 09$	$162, 39$
(G, L)	$44, 08$	0	0
(H, D)	$71, 07$	0	$82, 73$
(I, E)	$183, 86$	$72, 4$	$266, 09$
(J, A)	0	0	0
(K, F)	$51, 75$	$94, 27$	$46, 96$
(L, G)	$100, 42$	0	0
Total	$1338, 89$	$638, 42$	$1623, 82$

TABLE 1. Double track case, without including a lower bound

	Linear problem				
Corridors	Path	Total Trains	without Delay	% Delayed	% without Delay
(A, C)	1	103	0	100%	0%
(B, G)	3	73	30	59%	41%
$(9, F)$	5	55	0	100%	0%
(A, J)	7	0	0	0%	0%
(L, G)	9	100	19	81%	19%
(D, G)	11	60	0	100%	0%
(D, H)	13	1	0	100%	0%
(K, F)	17	52	0	100%	0%
(I, E)	15	184	30	84%	16%
(C, A)	2	86	0	100%	0%
(G, B)	4	34	0	100%	0%
$(F, 9)$	6	11	0	100%	0%
(J, A)	8	0	0	0%	0%
(G, L)	10	44	8	%82	18%
(G, D)	12	20	0	100%	0%
(H, D)	14	71	2	97%	3%
(F, K)	18	64	0	100%	0%
(E, I)	16	381	117	69%	31%
Total		1339	206	$84, 62\%$	$15, 38\%$

TABLE 2. Delayed trains for the test network

	Linear problem		
Corridors	App 1	LB App 2	UB App 2
$(9, F)$	$19,83$	20	51
(A, C)	38	$74,04$	$140,59$
(A, J)	16	16	16
(B, G)	$156,12$	4	4
(C, A)	$73,41$	$74,04$	$156,1$
(D, G)	$145,35$	23	$150,29$
(D, H)	12	12	12
(E, I)	$354,98$	$68,78$	$319,65$
$(F, 9)$	20	20	20
(F, K)	35	$81,88$	$187,76$
(G, B)	$69,32$	4	4
(G, D)	20	22	$149,29$
(G, L)	5	5	16
(H, D)	12	12	28
(I, E)	$160,18$	$69,78$	$265,68$
(J, A)	15	15	15
(K, F)	$77,42$	$81,88$	$47,37$
(L, G)	4	4	15
Total	$1233,61$	$607,4$	$1597,73$

TABLE 3. Double track case, including lower bound

	Linear problem				
Corridors	Path	Total Trains	without Delay	% Delayed	% without Delay
(A, C)	1	38	2	95%	5%
(B, G)	3	156	8	95%	5%
$(9, F)$	5	20	0	100%	0%
(A, J)	7	16	2	88%	13%
(L, G)	9	4	1	75%	25%
(D, G)	11	145	32	78%	22%
(D, H)	13	12	2	83%	17%
(K, F)	17	77	26	66%	34%
(I, E)	15	160	34	79%	21%
(C, A)	2	73	4	95%	5%
(G, B)	4	69	1	99%	1%
$(F, 9)$	6	20	0	100%	0%
(J, A)	8	15	0	100%	0%
(G, L)	10	5	0	100%	0%
(G, D)	12	20	0	100%	0%
(H, D)	14	12	1	92%	8%
(F, K)	18	35	2	94%	6%
(E, I)	16	355	131	63%	37%
Total		1232	246	$80,03\%$	$19,97\%$

TABLE 4. Delayed trains for the network considering lower bound

	Linear Problem			
Corridors	Lower Bound	App 1 without LB	App with LB	Using
(A, C)	38	$102, 71$	38	100%
(C, A)	38	$86, 34$	$73, 41$	52%
(B, G)	4	$72, 56$	$156, 12$	3%
(G, B)	4	$34, 42$	$69, 32$	6%
$(9, F)$	19	$55, 14$	$19, 83$	96%
$(F, 9)$	20	$10, 88$	20	100%
(A, J)	16	0	16	100%
(J, A)	15	0	15	100%
(L, G)	4	$100, 42$	4	100%
(G, L)	5	$44, 08$	5	100%
(D, G)	23	$59, 88$	$145, 35$	16%
(G, D)	20	$20, 3$	20	100%
(D, H)	12	$0, 6$	12	100%
(H, D)	12	$71, 07$	12	100%
(I, E)	16	$183, 86$	$160, 18$	1%
(E, I)	16	$381, 03$	$354, 98$	5%
(K, F)	35	$51, 75$	$77, 42$	45%
(F, K)	35	$63, 85$	35	100%
Total	332	$1338, 89$	$1233, 61$	$12, 33\%$

TABLE 5. Use of the network

Section	km	Section	km	Section	km
$(A,1)$	$4,2$	$(93,94)$	$4,7$	$(N6,70)$	$0,4$
$(1,2)$	$2,7$	$(94,95)$	$5,2$	$(70,69)$	$1,6$
$(2,3)$	$2,1$	$(95,J)$	$7,8$	$(69,68)$	$2,1$
$(3,4)$	$3,8$	$(A,65)$	$4,4$	$(68,N7)$	$0,5$
$(4,5)$	$5,1$	$(65,64)$	$7,8$	$(C,N3)$	$1,2$
$(5,6)$	$7,2$	$(64,63)$	$6,8$	$(N3,17)$	$0,8$
$(6,7)$	$8,4$	$(63,62)$	5	$(17,16)$	1
$(7,8)$	$4,9$	$(62,61)$	$3,4$	$(16,13)$	$3,5$
$(8,9)$	$2,3$	$(61,60)$	$4,6$	$(N3,N4)$	$0,5$
$(9,10)$	$3,7$	$(60,59)$	$3,7$	$(N4,15)$	$1,5$
$(10,11)$	$1,6$	$(59,58)$	7	$(N3,15)$	$0,5$
$(11,N1)$	$5,7$	$(58,K)$	$6,1$	$(15,51)$	$3,2$
$(N1,B)$	$6,3$	$(K,56)$	$3,6$	$(51,50)$	$5,7$
$(N1,12)$	4	$(56,N8)$	$2,3$	$(50,49)$	$3,1$
$(12,N2)$	$1,2$	$(N2,13)$	$2,2$	$(49,N9)$	2
$(N2,D)$	$1,7$	$(13,14)$	$2,6$	$(N9,48)$	$1,5$
$(D,52)$	$2,7$	$(14,N4)$	$2,1$	$(48,47)$	$3,1$
$(52,53)$	$1,5$	$(N4,E)$	$4,1$	$(47,F)$	$7,1$
$(53,54)$	2	$(E,72)$	$3,3$	$(F,46)$	2
$(54,L)$	$3,8$	$(72,71)$	$0,9$	$(46,45)$	6
$(L,55)$	$3,3$	$(71,N6)$	$1,3$	$(45,44)$	$4,2$
$(55,N8)$	$2,9$	$(N6,N5)$	$0,2$	$(44,43)$	$5,8$
$(N8,66)$	$5,5$	$(N5,73)$	$4,5$	$(43,42)$	$3,7$
$(66,67)$	$3,6$	$(73,74)$	$2,9$	$(42,41)$	$5,4$
$(67,I)$	$3,5$	$(74,75)$	2	$(41,40)$	$2,7$
$(I,N7)$	3	$(75,76)$	$2,8$	$(40,39)$	$5,8$
$(N7,85)$	2	$(76,77)$	$8,4$	$(39,G)$	5
$(85,86)$	$2,7$	$(77,78)$	$2,7$	$(G,38)$	$9,3$
$(86,87)$	$2,3$	$(78,79)$	$6,1$	$(38,37)$	$5,8$
$(87,88)$	$4,7$	$(79,80)$	$4,6$	$(37,36)$	$4,4$
$(88,89)$	$2,1$	$(80,81)$	$5,3$	$(36,35)$	$2,5$
$(89,90)$	$3,8$	$(81,82)$	$5,2$	$(35,34)$	$2,1$
$(90,91)$	$1,7$	$(82,83)$	$2,1$	$(34,33)$	$2,5$
$(91,92)$	$5,8$	$(83,84)$	$3,4$	$(33,32)$	$3,7$
$(92,93)$	$2,6$	$(84,H)$	$10,8$	$(32,31)$	4
$(31,30)$	$2,8$	$(26,25)$	2	$(21,20)$	$1,3$
$(30,29)$	$2,2$	$(25,24)$	$3,2$	$(20,19)$	$2,7$
$(29,28)$	$2,7$	$(24,23)$	3	$(19,18)$	$2,8$
$(28,27)$	$5,1$	$(23,22)$	$0,8$	$(18,15)$	$5,6$
$(27,26)$	$4,7$	$(22,21)$	$2,4$		

TABLE 6. Distance for sections

Arc	Traffic	Arc	Traffic	Arc	Traffic	Arc	Traffic	Arc	Traffic
$(N2, 13)$	391	$(38, G)$	153	$(8, 7)$	75	$(29, 28)$	25	$(86, 85)$	15
$(N7, I)$	390	$(37, 38)$	153	$(9, 8)$	75	$(28, 27)$	25	$(87, 86)$	15
$(E, 72)$	383	$(36, 37)$	153	$(N1, B)$	68	$(27, 26)$	25	$(88, 87)$	15
$(72, 71)$	383	$(35, 36)$	153	$(46, F)$	68	$(26, 25)$	25	$(89, 88)$	15
$(71, N6)$	383	$(34, 35)$	153	$(45, 46)$	68	$(25, 24)$	25	$(90, 89)$	15
$(N6, 70)$	371	$(33, 34)$	153	$(44, 45)$	68	$(24, 23)$	25	$(91, 90)$	15
$(70, 69)$	371	$(32, 33)$	153	$(43, 44)$	68	$(23, 22)$	25	$(92, 91)$	15
$(69, 68)$	371	$(31, 32)$	153	$(42, 43)$	68	$(22, 21)$	25	$(93, 92)$	15
$(68, N7)$	371	$(30, 31)$	153	$(41, 42)$	68	$(21, 20)$	25	$(94, 93)$	15
$(N3, 15)$	318	$(29, 30)$	153	$(40, 41)$	68	$(20, 19)$	25	$(95, 94)$	15
$(N9, 48)$	254	$(28, 29)$	153	$(39, 40)$	68	$(19, 18)$	25	$(J, 95)$	15
$(48, 47)$	254	$(27, 28)$	153	$(G, 39)$	68	$(18, 15)$	25	$(65, A)$	15
$(47, F)$	254	$(26, 27)$	153	$(9, 10)$	57	$(15, N3)$	25	$(64, 65)$	15
$(I, N7)$	238	$(25, 26)$	153	$(10, 11)$	57	$(D, 52)$	20	$(63, 64)$	15
$(N4, N3)$	218	$(24, 25)$	153	$(11, N1)$	57	$(52, 53)$	20	$(62, 63)$	15
$(13, N2)$	215	$(23, 24)$	153	$(N2, D)$	52	$(53, 54)$	20	$(61, 62)$	15
$(N1, 12)$	210	$(22, 23)$	153	$(N3, 17)$	52	$(54, L)$	20	$(60, 61)$	15
$(12, N2)$	210	$(21, 22)$	153	$(17, 16)$	52	$(52, D)$	20	$(59, 60)$	15
$(13, 14)$	210	$(20, 21)$	153	$(16, 13)$	52	$(53, 52)$	20	$(58, 59)$	15
$(14, N4)$	210	$(19, 20)$	153	$(56, K)$	50	$(54, 53)$	20	$(55, N8)$	15
$(72, E)$	183	$(18, 19)$	153	$(N8, 56)$	50	$(L, 54)$	20	$(N6, N5)$	12
$(71, 72)$	183	$(15, 18)$	153	$(A, 1)$	38	$(N4, 15)$	19	$(N5, 73)$	12
$(N6, 71)$	183	$(48, N9)$	123	$(1, 2)$	38	$(58, K)$	16	$(73, 74)$	12
$(D, N2)$	181	$(47, 48)$	123	$(2, 3)$	38	$(N7, 85)$	16	$(74, 75)$	12
$(17, N3)$	181	$(F, 47)$	123	$(3, 4)$	38	$(85, 86)$	16	$(75, 76)$	12
$(16, 17)$	181	$(15, N4)$	100	$(4, 5)$	38	$(86, 87)$	16	$(76, 77)$	12
$(13, 16)$	181	$(K, 56)$	98	$(5, 6)$	38	$(87, 88)$	16	$(77, 78)$	12
$(15, 51)$	172	$(56, N8)$	98	$(6, 7)$	38	$(88, 89)$	16	$(78, 79)$	12
$(51, 50)$	172	$(10, 9)$	95	$(7, 8)$	38	$(89, 90)$	16	$(79, 80)$	12
$(50, 49)$	172	$(11, 10)$	95	$(8, 9)$	38	$(90, 91)$	16	$(80, 81)$	12
$(49, N9)$	172	$(N1, 11)$	95	$(N3, C)$	38	$(91, 92)$	16	$(81, 82)$	12
$(70, N6)$	171	$(N3, N4)$	91	$(66, N8)$	35	$(92, 93)$	16	$(82, 83)$	12
$(69, 70)$	171	$(51, 15)$	88	$(67, 66)$	35	$(93, 94)$	16	$(83, 84)$	12
$(68, 69)$	171	$(50, 51)$	88	$(I, 67)$	35	$(94, 95)$	16	$(84, H)$	12
$(N7, 68)$	171	$(49, 50)$	88	$(N9, N5)$	35	$(95, J)$	16	$(N5, N6)$	12
$(12, N1)$	163	$(N9, 49)$	88	$(N5, N7)$	35	$(A, 65)$	16	$(73, N5)$	12
$(N2, 12)$	163	$(N8, 66)$	82	$(N4, E)$	28	$(65, 64)$	16	$(74, 73)$	12
$(14, 13)$	163	$(66, 67)$	82	$(E, N4)$	27	$(64, 63)$	16	$(75, 74)$	12
$(N4, 14)$	163	$(67, I)$	82	$(G, 38)$	25	$(63, 62)$	16	$(76, 75)$	12
$(F, 46)$	153	$(N5, N9)$	82	$(38, 37)$	25	$(62, 61)$	16	$(77, 76)$	12
$(46, 45)$	153	$(N7, N5)$	82	$(37, 36)$	25	$(61, 60)$	16	$(78, 77)$	12
$(45, 44)$	153	$(C, N3)$	75	$(36, 35)$	25	$(60, 59)$	16	$(79, 78)$	12
$(44, 43)$	153	$(1, A)$	75	$(35, 34)$	25	$(59, 58)$	16	$(80, 79)$	12
$(43, 42)$	153	$(2, 1)$	75	$(34, 33)$	25	$(N8, 55)$	16	$(81, 80)$	12
$(42, 41)$	153	$(3, 2)$	75	$(33, 32)$	25	$(55, L)$	16	$(82, 81)$	12
$(41, 40)$	153	$(4, 3)$	75	$(32, 31)$	25	$(L, 55)$	15	$(83, 82)$	12
$(40, 39)$	153	$(5, 4)$	75	$(31, 30)$	25	$(K, 58)$	15	$(84, 83)$	12
$(39, G)$	153	$(6, 5)$	75	$(30, 29)$	25	$(85, N7)$	15	$(H, 84)$	12
$(B, N1)$	153	$(7, 6)$	75	–	0	–	0	–	0

TABLE 7. Maximum possible traffic by section

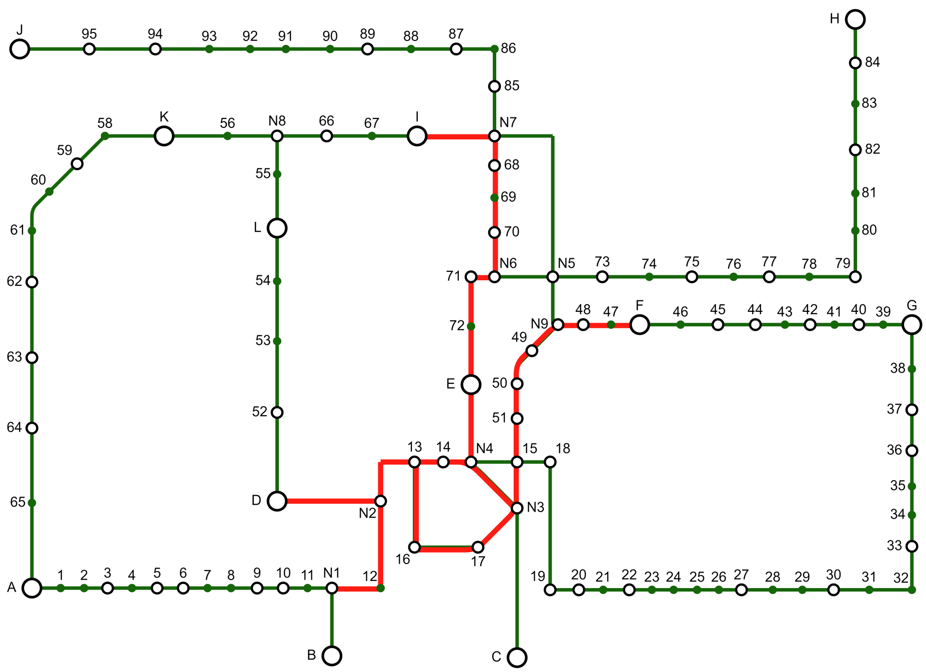

FIG. 2. Segments with higher possible traffic of Catalunya Rodalies Network